Plane talking

Mathematicians show you can't hear geometric shapes

You don't have to march to the beat of a different drummer to know that you can't hear the shape of a drum. So says a husband-wife mathematics team who, with a colleague, have solved a century-old problem that borders on the psychedelic: Can you hear geometric shapes?

No, say Carolyn S. Gordon, Ph.D., professor of mathematics, and David L. Webb, Ph.D., associate professor of mathematics at Washington University, and Scott Wolpert, Ph.D., assistant professor of mathematics at the University of Maryland. Employing a novel geometrical approach that uses generalized surfaces called orbifolds and a recent theorem of the mathematician Pierre Berger, the three have shown that two very differently shaped geometric figures drawn in a two-dimensional plane may produce precisely the same sound. Thus, even with a perfect ear, you could not determine from the sound alone which of the geometric shapes you were listening to. An orbifold is a surface containing a few "bad" points where the geometry appears folded or crinkled.

Imagine two drumheads with different shapes. They can be oval, triangular, pentagonal, whatever. If both drums are struck, you would expect that the difference in their shapes would cause them to emit different sounds. But Gordon, Webb and Wolpert have discovered two differently shaped regions in the plane that, when they are set vibrating, produce exactly the same set of frequencies, or overtones.

Why had people thought it possible to "hear" shapes? First, some earlier mathematical theorems suggested the intriguing possibility. And second, everyday experience indicates it. The idea is routinely used in industry to test equipment such as turbines, plates and other components of machinery subject to stress and fatigue. If a propeller blade develops a tiny hairline crack — disastrous if undetected — the change in the blade's shape is reflected by a change in the way the blade vibrates, and thus in its sound. And in the old days, experienced railroad workers often would walk down the lines, striking the wheels of a stationary train with a metal rod, listening for anything suspicious. The old hands knew what a "healthy" wheel sounded like, and a wheel with a developing crack did not sound the same as a normal wheel.

Gordon and Webb first announced their discovery in June 1991 at the Institute Fourier in Grenoble, France, and then in July 1991 at the National Science Foundation's Regional Geometry Institute at Mount Holyoke, Mass. Their work will appear in a forthcoming issue of the Annals of the American Mathematical Society. Their announcement created a stir in the mathematics community because it resolved a question, nearly a century old, that first came into consideration in the days of Lord Rayleigh, the brilliant English physicist who pioneered the theory of radiation.

Rayleigh's theory of black-body radiation involves a calculation of the energy at every wavelength, and it ultimately sparked the mathematical question. If the spectra — the sequences of frequencies — of two surfaces or regions in the plane are the same so that both surfaces resonate at the same frequencies, do they have the same area? The German mathematician David Hilbert, one of the greatest and most prolific mathematicians of this century, predicted that the question would not be solved in his lifetime. Two years later, in 1911, Hilbert witnessed his student Hermann Weyl prove him wrong when Weyl showed that if two surfaces produce the same sounds, then they have the same area. Later, two other mathematicians showed that if two plane regions produce the same sounds, then their peripherals must be the same. Thus, if two drumheads that produce the same sounds have the same area and perimeter, it was only natural to ask: Do they have the same shape?

In 1966, the highly regarded mathematician Mark Kac won the Chauvenet Prize, an award for mathematical exposition, for his celebrated article, "Can One Hear the Shape of a Drum?" which posed exactly this question. His paper reawakened interest in the question.

A snapshot study

Imagine two drums, perhaps not the kind that Charlie Watts or Bingie Starr are famous for, but two similarly shaped regions in a plane. They may, in fact, be triangular, oval or even irregularly shaped. The question Gordon, Webb and Wolpert and many mathematicians before them considered is: If you could hear the entire array of overtones produced when the drum is hit, and if your ear could discriminate perfectly between different pitches, could you from the sound deduce the shape of the drum? The vibration of a membrane is described mathematically by a partial differential equation called the wave equation. This equation involves a mathematical entity called the Laplace operator. The eigenvalues of the Laplace operator are numbers that mathematicians can associate with the Laplace operator; their physical interpretation is that they specify the overtone frequencies at which the membrane can vibrate.

Gordon and Webb began with a problem in partial differential equations and solved it by a geometric proof that is easily visualized. The mathematicians, in effect, imagined a snapshot of one of the vibrating drumheads at a single instant. The peaks and valleys of the vibrating drumhead at that instant form a sort of "warped membrane," as Webb puts it. The researchers then cut apart the warped membrane and reassembled the pieces to get another warped membrane — a snapshot of the possible vibration of the other drumhead. They took considerable care to ensure that the pieces fit together perfectly. The first step needed, says Ozawa, is an overhaul of the AFDC payment method, a federally mandated matching plan that also is used to determine Medicaid payments. Currently, each state first determines its own payment level by using a formula that relies on the state's per capita personal income. Once that figure is established, the federal government makes up the difference. The federal government matches 55 percent of the difference. Federal subsidies are determined as a percentage of the state's share of welfare costs, the federal share being 50 percent and the state's share being 50 percent. Yet the federal government generates the nation's children — and the country's economic future — by not enforcing legislation that equates federal AFDC payments. In addition, says AFDC children are being punished for the way society views their parents.

"It's an unfair policy and the biggest victims are our children," says Ozawa, a specialist in national social welfare policy.

In her paper, Ozawa contends that the federal government is not fulfilling its role of changing the nation's children — and the country's economic future — by not enforcing legislation that equates federal AFDC payments. In addition, she says AFDC children are being punished for the way society views their parents.
**Haute (Hot) Stuff**
is fashion show theme

The School of Fine Arts will hold its annual fashion show, titled "Haute (Hot) Stuff," on May 5 and 6 p.m. Friday, May 8, in the Missouri Historical Society's Jefferson Memorial Building.

The specially choreographed fashion show, complete with music and lights, features ballgowns, cocktail dresses, bathing suits, spring dresses, formal dresses, coats, suits, ensembles, sportswear, men'swear and collection items such as embellished tops. Leigh Singleton, associate professor of art and fashion design, says his 22 students (11 seniors and 11 juniors) created 175 "clothed figures" this year. A a clothed figure is a complete outfit, such as a jacket and skirt, rather than a single piece.

Design prizes will be awarded at the 5 p.m. show. Those prizes are being awarded by many companies: Anadul's Fabrics, Bernina of America, Dominic-Michael Hair Design Inc., Eunice Farmer Fabrics, Jackman's Fabrics, Kellwood Corp., Lord & Taylor, the Stanley Heller National Association of Men's Sportswear Buyers (NAMSBB) Foundation, and Winona's Fabrics.

The 8 p.m. show, which will be held in the Loebner Gallery of the Jefferson Memorial Building, includes a cash bar, a dinner buffet, and dancing to Orquesta Solucion Latina, a 15-piece salsa band. The show also features a preview of the Missouri Historical Society's exhibit, "From Carriage Trade to Ready-Made: St. Louis Clothing Designers, 1880-1920."

The school's annual fashion show is being held in conjunction with the society's exhibit on St. Louis fashion design, which runs from May 9 through the end of May.

Tickets to the 5 p.m. show and awards ceremony are $12.50 and tickets to the 8 p.m. show are $50. "Haute (Hot)" seats in the front row may be reserved for $100. All proceeds from the fashion shows benefit the school's annual fashion show, titled "Haute (Hot) Stuff," on May 9-16 at the Mallinckrodt Upper Level Drawing, taught by Washington's mathematics department in honor of their proof.

**Plane talking**

smoothly so that they could be reassembled into a snapshot of the other drumhead vibrating. The examples they produced settled Kac's question definitively: The sound produced by a shape is not distinctive.

Several things fell into place, paving the way for Gordon, Webb and Wolfpert to arrive at their solution. The first involved a theorem of the Japanese mathematician T. Sunada. The theorem provided an algebraic framework for the "reassemblage" of the pieces of the warped membrane.

Mathematicians, including Robert Brooks of the University of Southern California and Swiss mathematician Peter Buser, already had used Sunada's work to construct surfaces that produce the same sounds. However, these surfaces did not solve Kac's problem because they are two-dimensional surfaces that exist in higher dimensional spaces, but not inside the two-dimensional plane, so they could not be used as drumheads.

"Sunada's theorem intrigued Gordon, Webb and Wolfpert, but it was inapplicable to the geometric regions in the plane that they wished to study. Then, along came Pierre Berard's proof that Sunada's theorem works in a context that would include the orbifolds the three mathematician team intended to explore. Berard's proof showed how to take the waves from one drum and "transplant" them to the other."

"Berard's proof was an invaluable idea, opening the door for the use of orbifolds," says Webb.

"An orbifold arises, Webb explains, when an ordinary smooth surface is folded or crinkled in a certain way. While it looks like a smooth surface at most points, there are a few "bad" points where, for example, it resembles a fold in a sheet of paper. Gordon and Webb constructed their examples by folding Buser's examples of surfaces to produce orbifolds. They used Berard's version of Sunada's theorem to show that because the time-frozen waves could be reassembled, the frequencies produced by both orbifolds were the same. Finally, they built models of the orbifolds to see that they were shaped very differently.

**Building paper models**

Because the researchers needed to show that the frequencies of the two drums are identical, a computer, which can only calculate very close approximations to the frequencies, was of no use. And while the researchers devised a complicated abstract, theoretical proof, the essence of their work is easy to see in simple construction-paper models Gordon and Webb built in the living room of their house.

"One of the unusual and refreshing aspects of this project is that such concrete models can be constructed," Webb says. "One of Carolyn's major contributions to the question of how shape is related to sound is her discovery, with Edward N. Wilson (dean of Washington University's Graduate School of Arts and Sciences), of whole families of higher dimensional surfaces that sound the same. But these examples are all in six or more dimensions, so you can't hope to build models. What was fun about this project is that we could sit in our living room and, beginning with some elementary algebra called group theory, build paper models of surfaces. The point of building an was an important aspect of the project." Indeed, in addition to laying to rest an old math question, what may be most impressive about the Washington University team's work is its simplicity, says Dennis DeTurck, Ph.D., professor of mathematics at the University of Pennsylvania.

"Gordon and Webb have taken an essential mathematics problem, around which an incredibly abstract mathematics machinery has been built over a hundred years, and solved it so that a sophomore calculus student can understand it," DeTurck says. "Their proof will be a part of mathematics textbooks and monographs for years to come."

— Tony Fitzpatrick

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**Students exhibit works at local gallery**

Juniors, seniors and graduate students in painting in the School of Fine Arts will exhibit their works in a free show, titled "Pentimenti," May 9-16 at the Utopia Loft Gallery, 3524 Washington Ave.

The exhibit will feature between 16 and 18 works from a class, titled Upper Level Drawing, taught by Phyllis Plattner, a local painter and visiting associate professor of art. Plattner specializes in art history. Her work is in major collections nationwide.

"Pentimenti" is an Italian word for repentance. In art, the word is used to describe a painting that occurs when the top layers of a painting fall off, revealing earlier versions of the work to be seen. Pentimenti reveals how a painter changes his or her mind during the artistic process.

The students chose this for the title of their show as a way to convey how their ideas and styles evolve. The exhibit will be open from 11 a.m. to 4 p.m. Tuesday through Sunday. For more information, call 531-5515.
Christopher T. Walsh, Ph.D., chairman of the Department of Molecular, Cellular and Developmental Chemistry and Molecular Pharmacology at Harvard Medical School, will present the 15th annual Oliver H. Lowry Lecture in Pharmacology, titled "Molecular Studies on Prolyl Isomerases," will be at 4 p.m. May 14 in the Carl V. Moore auditorium in the University of Washington, Seattle, and will be followed by a reception. For information about the lecture, call 362-7053.

Correction

Due to computer problems, several errors appeared in the April 23 edition of the Daily Record.

On Page 3, three sentences were omitted from the last paragraph of the story "Dumpy you up - Growth hormone no magic bullet for building muscle." The paragraphs should have read: According to Yasheshki's findings, even if former NFL defensive lineman Lytle Allred had used actual growth hormone, it probably didn't enhance his muscle growth. "It's not because of the muscle factor," he says. "Instead, we believe the secret is growth hormone's ability to reverse the cellular aging process, and that process is not under the hormonal control." These two hormones have provided a great number of insights about the regulation of metabolism in a variety of differentiated cell types.

For information about the lecture, call 362-7053.

Introductions to new faculty

The Record is running a series profiling new faculty on the Hilltop and Medical campuses.

Diane E. Beals, Ed.D., assistant professor of education, comes to Washington from Harvard Graduate School of Education, where she served as teaching fellow while completing her doctorate in human development. She received her bachelor's degree in general science and elementary education, cum laude, in 1979 from Seattle Pacific University. She received her master's degree in developmental reading in 1984 from the University of Washington, Seattle, and her doctorate in 1991 from Harvard.

She has many research and teaching interests, including child language, development of discourse skills, and individual and cultural differences in language and literacy development. She has published many articles, including "Stories From the Classroom: Rate of Reading," in The Norma Lowry Memoir Fund Prize, open to all students at Washington University, was won by Allison Funk, assistant professor of English at Southern Illinois University College student, for his work, titled "Orders." Sarah Beck received honorable mention for "Vikings." All poetry entries were judged by Allison Funk, assistant professor of English at Southern Illinois University. The Roger Conant Hatch Fund Prize, open to all Washington University undergraduates, was awarded to Michael Sinclair, a University College student, for his poetry collection "Drive-In West." The Norma Lowry Memoir Fund Prize, open to all Washington University students, was won by Camelia Ibele, a graduate student in the Writing Program, for "Elephant Rocks." Xiaolong Qiu, a student in the joint Ph.D. Chinese and comparative literature program, received honorable mention for "Li Shuangzi's Case of Distress." The Academy of American Poets Prize, also open to all students at Washington University, was won by Anne Marie Cusack, a graduate student in the Writing Program, for her short story, "Salt Mother." All entries in the contest will be awarded at the English Department's final meeting of the academic year.
Thursday, April 30
9:30 a.m. Dept. of Internal Medicine Grand Rounds. "Emotions and the Heart," Francis Vance Visiting Professor, "The Many Faces of Depression." Vicki C. Spring, M.D., prof, of medicine, director, Psychosomatic Medicine, the Cardiovascular Health Research Program, San Francisco Medical Clinic. Coplon Aud., 450 Audubon.


4:30 p.m. Dept. of Physical Therapy Seminar, Dr. Steven J. Kohne Leadership. "Rehabilitation of Balance Disturbances," Dr. Steven J. Kohne, assoc. prof, and head, WU Dept. of Physical Therapy. Richard P. Cendrowski Auditorium.


8 p.m. Dept. of Music Electronic Music Presenting Center Presents a Seminar, "Document Processing." Karl H. Gustafson, Ph.D., WU grad, of computer and information science. WU Music Center. Room 202, 1100 Hilbert Hall. Cost: $10 for WU faculty and staff. For more info., call 935-5889.

Friday, May 1
7:30 p.m. St. Louis Psychiatric Center. "An Evening with the Co-Founders of the Paris/Ohio State Institute and the Art Museum." For more info., call 727-0027.

Calendar Deadline