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A Hierarchical Model of Living Systems: A Unifying Concept in Biopsychosocial Medicine

John Mosby Grant

A HIERARCHICAL MODEL OF LIVING SYSTEMS:

A UNIFYING CONCEPT

IN

BIOPSYCHOSOCIAL MEDICINE

BY

JOHN M. GRANT, M.D.

1997

John Mosby Grant, M. D., September 1996

A HIERARCHICAL MODEL OF LIVING SYSTEMS:

A UNIFYING CONCEPT

IN

BIOPSYCHOSOCIAL MEDICINE

Consisting of:

- **Hierarchically ordered descriptions**
- **Means of classifying such systems**
- **Predictions based on this scheme**

BY

JOHN M. GRANT, M.D.

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This book is dedicated to the
important Grant women in my life:

Natalie N. Grant
Margaret T. Grant
Dionne D. Grant
Natalie T. Grant

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These individuals are devoted friends who have helped in one aspect or another in the production of this monograph. They have encouraged me to finish the paper, they have edited, typed, polished the writing, contributed content, funding and managed printing, binding and distribution. This bound volume simply would not have come to completion without their help. I am deeply grateful to them.

Penelope and John Biggs,
Ellie and the Rev. William Chapman,
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Margery Dodson Imster also to honor her brother Roland W. Dodson, Jr. a friend,
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Barbara and Milton L. Rand, Jr.,
Kyra and Lynn E. Stuart,
Judy Weissman.

John M. Grant, M.D.
St. Louis, Missouri
February, 1997

PREFACE

When I was young and innocent, a freshman premedical student at Princeton University, I found myself the very first year in a department both fascinating and challenging, perhaps even more so than premed. Furthermore, it was the best department of mathematical physics in the world. I could not give up medicine, but perhaps I could use mathematics in my study of biology and medicine. If mathematics is the preferred language of science, and biology is certainly a science, why not mathematical biology?

By my third year in physics, I had found a suitable subject. Raphael Lorente de No, at the Rockefeller Center, had written a tome on mathematical models of nerve function. I felt that I could expand on his work by using electronic models and indeed did for my senior thesis. I was lucky enough to do this under the guidance of Drs. John Wheeler and Donald Hamilton, two distinguished mathematical physicists.

At Washington University Medical School, I had the opportunity to work on real nerve fibers in the lab of Dr. Joseph Erlanger, who had received the Nobel Prize for this work just a few years earlier. In England, Hodgkin and Huxley were successfully grinding out their mathematical equations on nerve function. Mathematical biology was here to stay.

However, as I progressed up the medical curriculum, things got harder again and I found less and less mathematics used in describing things biologically. My dream eventually began to fade, particularly after I got more and more involved in the care of patients. However, a spark remained and, as will be seen in the text, such regularly occurring phenomena kept popping up throughout most living systems that I could not help wondering whether they might be following the laws of some mathematics. For instance, living things could be divided into smaller living things, forming levels or organization that had a hierarchical structure. These were whole-part hierarchies since the parts fit into the whole and in turn the parts became wholes at a lower level. It began to appear as if all living systems had a hierarchical structure.

But what kind of mathematics would do the job? Many of the most important biological properties are difficult to measure using the integers or the reals. But they can be measured using the binary variables (P) and (P') of various biological properties. For instance, the presence (P) or absence (P') of the properties' growth, replication, interaction, boundary, self support, or self movement may all be easily measured, while the amount may be considerably more difficult. In addition, one may use this same mathematics in dealing with one, hundreds, or trillions of objects at the same time, a practice frequently necessary with biological objects. This, however, cannot be done with all mathematics. Thus discrete set or Boolean algebra might be a suitable mathematics to begin with because it has the additional advantage of being a rather simplified version of regular algebra.

Over the next many years, I worked between patients on my dream, assisted by some graphics which functioned as the equivalent of architectural plans, both of the elevation and floor-plan variety. They lay between real living systems and the mathematical model I was developing. In describing any particular living system, I had to decide how many levels of organization I would tackle, what properties I would look at, and in how much detail. This description was of interest in that it put together vertically biological properties that are usually related horizontally. However, when a number of similar living systems are described so, the situation becomes much more interesting. Now one can begin to classify these systems, often in quite fine detail and quite economically.

The fruit of most deductive systems lies in their ability to make predictions, and frequently these are related to classification systems. Recall that Mendeleev's classification of atoms led eventually to some of our most important theories in atomic physics. In a similar fashion, group theory led to a classification of elementary particles from which the omega (-) particle was predicted.

By now I was beginning to feel pretty happy with "hierarchy algebra," as I fondly called my little monster. Others, however, seemed to think of it as just a monster. I was having difficulty finding biologists who knew set algebra and mathematicians who knew biology, and those who knew both were too busy with their own work to learn a new dialect of Boolean algebra. And besides, who would expect to learn much from a practicing physician?

I don't know whether it bears any mind-body relationship or not, but it was not too long after this that I developed the first symptoms of what I later discovered was metastatic lung cancer to the bone (yes, I had been a smoker). I remarked one day to my close friend Jim Knight that I had one regret in dying and that was that I would never get to see my book published. "But, yes, you will," said he. About the same time, I received strong encouragement from Dr. George Engel and Dr. Robert McDowell to make a real effort to get my monograph printed. Within a matter of two months, it had transformed into this.

I am sending this to my friends and colleagues, as well as scientists who I think are interested, or should be interested. I will also be sending copies to medical school libraries, as well as others, and I hope to get a spot on the Internet. I think that if even a few find this of interest and carry on, the effort will have been worthwhile. I hope that those who find it usable will do so.

John M. Grant

March 1997

FOREWORD

No one could be less qualified than I am to comment on the mathematical details of this monograph. I never went past trigonometry at University City High School. I first read this work in 1993 simply as an informal editor, in order to fix up wayward sentences and rein in straying punctuations. I had a lot of time on my hands, for even Jack Grant had not been able to prevent me from being sick with metastatic breast cancer. After a particularly nasty reaction to chemotherapy I was confined to my bed and thought I could make myself minimally useful by going over this work, I became more and more interested as I read it, and, in spite of my limited understanding, felt that it was a noble attempt to offer a new tool for scientific and medical thought.

For about four hundred years intellectual movement has been going against hierarchy, order, inter-relatedness, in the West. No one could seriously want to return to the medieval ways of thought that Shakespeare mocked in *Richard II*, the belief that the heavens and the earth were both so carefully planned by God that all of nature would conspire to protect His anointed king. Neither God nor nature did a thing to protect Richard from usurper Bolingbroke, and over the next centuries many old ideas died. People ceased to believe in the parallel structure of the body and the earth that made rivers like veins, hills like breasts, and of the body and society, that made the king like a head, the working classes like the digestive system. By the eighteenth century even Samuel Johnson, hardly a radical, spoke contemptuously of the old great Chain of Being as Pope presented it in *An Essay on Man*. I think we would all agree with Johnson that if the world were fitted together with absolute interdependence the extinction of even one species would doom us all. We have, unfortunately, exterminated many species, and the world goes on. And I do not think many Americans would want to return to the mystically hierarchical political system that made ordinary people lowly subjects of a divinely ordained king.

But somehow the urge to take apart, to level, to separate gained a lunatic momentum of its own, and in the last generation we have seen some horrifying results. In my own field, English literature, the loathsome crew who proudly call themselves "postmodernists" and "deconstructionists" have denied the existence of authors, characters, stable selves, meanings of any kind in order to make us all--writers and readers alike--passive victims of linguistic patterns, "fields of power," or other monstrous constructions of their own invention. Any assertion of belief in the integrity of the person, or the necessity of moral systems, of the goodness and beauty of a work of literature, is seen as an act of complicity with racism, fascism, sexism, and imperialism. Those of us who still believe in wholeness and order of any sort have been banished to the intellectual boondocks.

Though medicine, as I have experienced it, remains a benign field (in contrast to English), surely Jack Grant is correct that doctors have lost an essential element of knowledge by shutting out much information about a patient's family and human situation. In my own case I feel--though I will never be able to prove--that a cancer which seemed to have been wiped out by surgery returned with a great power because I was surrounded by cruel colleagues. After one particularly crushing insult--I was told that I was going to be shut out of the graduate program--I fell into a state of despair. The metastatic cancer was discovered about nine months later. Unfortunately, once a cancer has spread, a change of circumstances and attitude do not cure it. As Voltaire said, prayer and arsenic together can kill rats. Pleasant thought and chemotherapy can beat back tumorous cells. Chemotherapy alone might work, but I certainly believed Jack when he told me, "If you give up, you will die. Put money in Growth Funds to show that you intend to live." And here I am, safely back in St. Louis; my Dreyfus Fund and I are both doing well. Without a doctor like Jack, I do not think I would be here.

But more than any individual, more than any field of study, our society itself is suffering from the loss of the old belief that wholeness and hierarchy are part of a natural order. We did not evolve to be discrete, independent beings, with discrete, independent organs. No primate is capable of growing up without parental guidance, and we all know what parts of town no sane person would venture into because the streets have been taken over by young men for whom no parental guidance, no community bonds exist. Words like "respect for your elders" sound as dated as "The Great Chain of Being." We are all paying the price for the extreme individualism that violates the evolutionary facts of our humanity.

But I do believe that more than one humane voice is offering resistance. We do not need the hierarchies and wholeness that the Rush Limbaughs of the world would impose upon us; we do need what other brave and lonely souls are urging. Oliver Sacks will not let us forget the mysterious power of the soul that continues to exist in people with bizarre symptoms of brain damage. E. O. Wilson is demanding that we remember that we and the rest of the animal world really do need something not altogether different from that old "Great Chain of Being"-- a diverse and healthy biosphere. Kirkpatrick Sale has asked us to become Dwellers in the Land, aware of the realities of bioregions. Robert Bly has condemned the particularly American form of social leveling that has resulted in a "sibling society" where no one respects anyone as a figure of authority. I think that Jack Grant belongs in this group, for he is offering a scheme with which people can talk clearly and rationally about how the living world fits together in a hierarchical whole that cannot be ignored or denied.

Jack has spent his life as a healer of individuals. I do not know how those of us who have been his patients (thirty-three years, in my case) will go on without him. Somehow we will have to. I hope that his monograph will allow him to continue his work as a healer, by letting his voice live on. I hope that doctors and scientists will receive this final gift from someone who has succeeded in seeing life steadily and seeing it whole.

Judy Weissman

St. Louis, Missouri

February 4, 1997

University of Rochester Medical Center
Department of Psychiatry
Division of Behavioral and
Psychosocial Medicine

January 31, 1994

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With no little trepidation the day before yesterday I picked up "Hierarchy Algebra" (now titled *Hierarchical Model of Living Systems*). Barring the distractions of everyday living, I did not put it down again until I read your final words: "- - - - We now have a highly selected hierarchically described and organized model of the living system we are investigating."

If I may mix metaphors, I am stunned and exhilarated at what you seem to have accomplished. For the first time you have provided me with and overarching model that accommodates the revolutionary significance of hierarchy, at least for me. What the afterthought refers to is my personal experience in reading what you have to say.

Journeying through this new edifice, I literally found in it a home for all that I have learned over virtually a life time but never have been able to express formally or informally beyond my strong intuition. I have never been able to use my two hierarchy diagrams, which I adapted from Paul Weiss, without feeling frustrated at how feebly they conveyed the basic notion of hierarchy, and my inability to do any more than add whatever examples happened to come to me at the spur of the moment. I have been operating primarily out of intuition, unable to grasp, much less explicate a conceptual structure which everyone could examine, share and critique. That is what you have provided, and it matters not a whit whether this or any other reader can literally follow the mathematics.

We need only your account of how one facile in that branch of knowledge would do so. You surely misstated your intentions when on page three you wrote: "If the reader knows none of these languages (the dialect of abstract Boolean algebra), they should venture no further." If you meant by that "no further in the book," I almost stopped at that point. Actually, what the reader needs to know from the beginning is that no formal personal knowledge or facility on the part of the reader is necessary, since you will make clear how the mathematics is used as a means of moving back and forth between the concrete and the abstract. That is something you do very well in the text.

A much more important point,with respect to qualifications for readership, is your reference to what is peculiar as to how physicians must work as scientists, namely vertically as well as horizontally. It is no accident that it is within medicine that we see the main interest in evolving this new paradigm, however inconspicuous it appears to be at the moment. Rene Dubos pointed out years ago that medicine by its very nature was from the beginning the origin of human science, at least to the extent that physicians (even healers) recognized that they were uniquely involved in studying other human beings and made efforts to systematize the procedures and the information and knowledge achieved thereby.

Your personal history and mine are not altogether dissimilar on that score. We both were raised literally from birth in a medical environment, and we each had brothers who followed us in the same direction. I like to make this point by referring to my "eighty years in medicine."

We both seem to have become involved with "science" very early. And we both floundered

We both seem to have become involved with "science" very early. And we both floundered about trying to reconcile what we were led to believe science encompassed with what we were actually required to deal with as physicians taking care of patients. If you are good enough to credit me with introducing you to the concept of hierarchy, I am indebted to you for helping me to make sense of what it was I introduced you to.

Obviously this work to which you have been devoting your life must get into print, and it is in that effort that I would like to help if I possibly can. I refer both to finding an interested publisher and to stylistic and compositional issues bearing on the attractiveness of the final product to a broad readership, including physicians, scientists, philosophers, and patients, the latter referring to everybody else. Note, I did not specify mathematicians, which is not to imply that their expertise is not critical, but that the book is not primarily intended to be a contribution to mathematics, however much it may turn out to be.

What I would like very much to do is talk to you about this at our mutual convenience, but as soon as possible because with each passing hour my clear recollection of what I just read and what my reactions were fade.

Actually I will try to call you even before you get this letter, the preparation of which at least has me oriented as to what some of the issue would be for me to discuss.

Sincerely,

George Engel, M.D.
Professor Emeritus of Psychiatry
Professor Emeritus of Medicine

Editor's note:

Drs. Grant and Engel met in March of 1994 . As a result, significant modifications were made to the monograph so that some of the page references in Dr. Engel's letter above no longer correctly apply.

Department of Mathematics

Comments on *The Hierarchical Structure of Living Systems* **by John M. Grant**

In my attempts to get an informed mathematician's opinion on the manuscript, I was unable to obtain guidance from algebraists, who would seem to have the most expertise in Boolean Algebras. Fortunately, I was able to enlist the help of a fine analyst, Richard Rochberg, whose research area involves some aspects of interacting hierarchies. After Dr. Rochberg had studied the manuscript, we discussed it for about an hour. This is a summary of that discussion.

There is an increasing interest in the study of hierarchies in various fields, each attacking the problems in its own way and for itsown purposes. For example, computer scientists working on hierarchical data bases must grapple with problems quite similar to those considered by Dr. Grant. The Grant manuscript takes an abstract, subject-free approach, using boolean algebra not merely in a purely descriptive manlier, but using algebraic properties, particularly distributive laws, to generate new relations. That is, despite the title and the numerous references to biological systems, the monograph in fact shows how to encode levels and interactions for any hierarchical system (which takes in a tremendous amount of ground). Its advantage, in biology and elsewhere, is to show how describing hierarchical systems using boolean algebra provides a language and method for tracing properties within the system. In this respect the discussion of psychosomatic illness was enlightening.

It will probably take the medical world a while to see the advantages of this approach, but in some form it seems bound to come. It would be a pity if Dr. Grant's work were not available to others. The monograph testifies to deep insights coupled with very difficult technical detail. It is clearly a labour of love on which a staggering amount of labour has been expended—work that would be very dif'ficult to duplicate. Thus we feel that it would be highly desirable to arrange for its publication, recognizing however that its austere style places heavy demands on the reader; it is not likely to have high sales in the foreseeable future.

Before publication, the manuscript should be routinely copy-edited. It would also be highly desirable to have someone go over the technical calculations, line by line, to eliminate possible typos or other errors. That will be a daunting task, but whoever does it will have a much clearer picture of the work, and might even be in a position to make some expository improvements.

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I) THE INDUCTION

A) 100 YEARS IN THE PRACTICE OF MEDICINE.

There have been more changes in medicine in the last 100 years than during any other comparable period in history. Most of these have been additive and positive, but some have been subtractive and negative. Perhaps the single most important change has been the explosion of information. For every journal one hundred years ago there are at least one hundred now, and for every book, a similar number. However, this information has relatively little overall organization, and to some extent science has crowded out the art and not always replaced it. Few have lived to witness these changes, but some medical families have and I have lived in one such. Although what is to follow is only one instance, it illustrates some of the points that are to be made in this book. Below I have given a brief biography of three generations of Grant physicians along with a summary and comment on some changes that have taken place.

1) The Family Physician

John M. Grant (I) graduated from The Missouri Medical College in 1889 and went into general practice the same year. He married four years later and his eldest son, Samuel B. Grant, was born in 1896. Although I never knew my grandfather (he died from overwork soon after the flu epidemic of 1919), we were bathed in stories about him, a few snippets of which I will relate here. As were most doctors of this time, he was a solo practitioner, both physician and surgeon. He was first and foremost a family physician, taking care of not only the nuclear, but also the extended family. In fact, family meant extended family in those days. He delivered babies, cared for children, doctored adults, and did surgery when this was called for. I still see as patients some of the babies that he delivered.

My father (Samuel Grant) used to drive him as he made his many house calls. He loved to tell us how his father would step out of the car when it was half a house away, using this motion to propel him to the front door. House calls (and office visits) usually involved more than one member of the family, and it was this fact that explained how not infrequently he would see up to 100 patients a day! His cars, one of which was a Stutz Bearcat (apparently sporty cars had appeal then also), had to be kept in a heated building so that they would start in the morning.

Not only was he involved with patients and families, he was very active in the St. Louis community. Public health was in the forefront then. For a while he was president of the board of education. All school flags hung at half mast when he died.

2) The Specialist

Samuel B. Grant was born in 1896, graduated from Washington University Medical School in 1920, spent three years of residency at the Brigham and Hopkins, and started practice in 1924. He married nurse Natalie Neville in 1926 and had his first two sons, John M. and Neville Grant, in 1927 and 1928 respectively. He was an internist and diagnostician and started out initially in solo practice. However, he soon formed a group which later became one of the largest groups of internal medicine physicians in St. Louis.

As an internist he took care of individuals and their "innards." One mind could encompass most of the knowledge of internal medicine in his early years, and for a number of years he taught the whole lecture course in internal medicine at Washington University Medical School. Although he

cared for nuclear families and made house calls, he felt the tug of specialization and started one of the first EKG laboratories west of the Mississippi. He also serendipitously happened upon respiratory alkalosis and tetany, a subject he was the first to describe and on which he published a number of articles

Sam Grant understood not only the science of medicine but also the art. He knew the family, had experienced his patient's past history, knew how to listen, and when he walked into the room, people felt better. My brother and I, who both went into practice with him, had no finer mentor.

3) The Subspecialists

John M. Grant III and Neville Grant. The former went to Princeton University, where he majored in mathematical physics. He then went to Washington University Medical School, graduating in 1954, had a medical residency and psychosomatic fellowship at Strong Memorial Hospital in Rochester, N.Y., and started in the practice of medicine at the Grant Medical Clinic in 1959. The latter went to Yale and then Columbia P & S, graduating in 1954. He spent his residency years at Barnes, Grace New Haven, and Stanford, and joined the group in the practice of internal medicine and endocrinology in 1960. The Grant Clinic, by now 20 years old, was taking in primarily subspecialists, although everyone practiced general internal medicine. Young physicians got their start at this time by going in with an established physician or practice. With the coming of Medicare and more widespread health insurance, however, new recruits found that they could quickly build up a practice, often in their subspecialty. Surgical consultations began to skip over the internist, going directly to the subspecialist. With easy patient transportation and nontransportable technology, the house call became uncommon.

Previously physicians had experienced the immediacy of their patient's family and past history. Now one had to depend almost entirely on a verbal account, the history. And even the history was being crowded as new technology sometimes made it seem irrelevant. With more and more knowledge about narrower fields, the subspecialist often would respond by saying the patient's problem was not in his or her field. The organ system, rather than the patient, became the focus of concern. However, because of the rapid advance in knowledge, the subspecialist sooner or later found that his field had outpaced him, and as he held on to more old patients he became more and more a generalist.

4) Some Observations

I would like to conclude this section with some overall observations on medical changes of the last 100 years. To present these changes more effectively, I will use an analysis approach which I will briefly define. According to Webster, analysis is the "separation of anything into component parts or elements; also, an examination of anything to distinguish its component parts, separately, or in their relation to the whole." On the other hand, synthesis is the "composition or combination of parts, elements, etc., so as to form a whole."

One of the major changes that have occurred over the last 100 years has been in the analysis of living systems. The organism which had previously been analyzed into organ systems, organs, tissues, and cells was now being broken up into organelles, macromolecules, molecules, and atoms. There was an explosion of knowledge in subcellular medicine and biochemistry.

Another major change has been in the change from family physician to specialist and later to subspecialist. This also followed the whole-part analysis. That is, family medicine went to specialty medicine (internal medicine, surgery, OB-GYN, and pediatrics), then to subspecialty (organ system) medicine (dermatology, gastroenterology, etc.), and finally to organ medicine (hepatology, cardiology, etc.). This progressive narrowing of fields also led to an exponential growth of knowledge and problems.

But not only were fields and physicians broken up, so was the patient. The "cared for" has gone from the family to the individual, to the organ system, even down to the organ. Not infrequently patients have felt that their physician was more interested in their intestinal tract than he was in them!

With these changes, there has also been a change in the doctor-patient relationship. It has become increasingly difficult for one doctor to know about all his patient's systems and subsystems. The patient is increasingly parceled out to different physicians for different problems, and often the patient is not sure who his physician is. In the same way, frequently it is difficult for the physician to talk with his patient of anything but his own system of expertise. Doctors even have difficulty in speaking with other doctors, the language often being so different in different fields.

As the patient and doctor have changed, so have the instruments of observation. For years, the most important tools of the physician's trade were his eyes, ears, and hands. This was all right when we dealt with only the upper levels of organization, but as we worked down we needed X-rays, electrocardiograms, microscopes, electron microscopes, test tubes, auto-analyzers, etc. In this process, there was a tremendous change in scale. Our object of concern went from being meters in size to centimeters, to microns, and finally to angstroms.

There were huge advantages to medicine in this kind of analysis. By breaking something into parts, one may be able not only to understand the whole better but also to focus down and direct one's energy, hopefully more effectively. But there are also disadvantages to analysis. Understanding the parts and their relationship to one another may be **necessary** in understanding the whole, but it is not **sufficient**, and there is a tendency to think it is sufficient. The levels above the parts play an important role in constraining everything below. In focusing down on the trees one loses sight of the forest. Another problem that arises in analysis is fragmentation of fields, the resulting language, and later interface problems.

Clearly, it would seem desirable to analyze and synthesize at the same time. That is, rather than working "down" (analysis) and then back "up" again (synthesis), might not one work from the side? I will show later how this is done.

B) THE METHOD AND THE INDUCTIVE-MODEL-DEDUCTIVE SCHEME

The method to be followed in this paper is called "abstracting the elements." It leads logically to what I have called "hierarchy algebra," a mathematical instrument similar to Boolean or set algebra. This approach has been chosen not only because it came naturally, but also because of an analogous relationship to an approach used in mathematical physics to investigate physical systems. This has been called "the method of elementary abstraction."

1) The Method of Elementary Abstraction as Used in Physics

Lindsay and Margenau describe this method well in their book *Foundations of Physics*. It is instructive to quote them:

> Perhaps we can best introduce the subject by quoting from a celebrated parable of the philosopher Schopenhauer. 'Two Chinamen traveling in Europe went to the theater for the first time. One of them did nothing but study the machinery, and he succeeded in finding out how it worked. The other tried to get at the meaning of the piece in spite of his ignorance of the language. Here you have the Astronomer and the Philosopher.' Now this may be somewhat rough on the philosopher, but everyone at once recognizes the profound distinction in method that is implied. A certain group of sense impressions were experienced

> by the two observers, who sought to describe the experience by two totally different ways: the one tried to appreciate the experience as a whole: the other, foregoing this, picked out of the whole one small part which he thought he might understand and successfully describe. It is this process of abstraction from the totality of physical phenomena which has undoubtedly been a leading feature in the success of the physical theorist. It is precisely this method of abstraction pushed to its logical conclusion that leads to the use of the differential calculus in physics. We shall call it the method of elementary abstraction.

They then go on to give an example of how a physicist would use elementary abstraction in investigating fluid motion:

> In his imagination he visualizes a large quantity of fluid. He then abstracts from the whole a very small volume element for his special consideration, and he considers its behavior over very short intervals of time. He makes the assumptions that the **elements** and intervals may be symbolically represented by mathematical infinitesimals. The symbolic expression of his further hypotheses concerning the behavior and properties of the fluid element will then contain differentials and derivatives, and hence will be a differential equation.

Lindsay and Margenau go on to say:

> Now that we have our differential equation, what shall we do with it? From it we can derive new equations which are in the nature of physical laws describing possible laboratory operations. Just how is this transition carried out? The mathematical process of passing from the differential equation to the physical law is known as integration. From the physical point of view the word almost literally conveys the meaning of the method, for just as the differential equation is a symbolic description based on the method of elementary abstraction, the resulting so-called solution is an algebraic equation describing the large-scale operations in the laboratory. Hence in a very true sense the passage from the differential equation to its solution involves a symbolic integration of the small-scale phenomena into large-scale phenomena.

2) The Method of Abstracting the Elements Used in this Book

The method I use is in many ways quite similar to the method described above. However, **the elements are quite different, and therefore lead to a different mathematical instrument, finite set algebra.** The elements in the physical example are infinitesimally small while, as will be seen, the elements in living systems are members of sets. They are wholes, quite discrete, and vary in size from the very small to the very large. Let me then proceed with the method of abstracting the elements. But first I would like to give an overview, a schema of this inductive-model-deductive process.

3) An Overview of the Inductive-Model-Deductive Scheme

Briefly, the inductive-deductive scheme divides itself into three sections (see Figure 1).

1) In the **induction** I have used the method which I have called the method of abstracting the elements. It begins with concrete living systems (wholes or elements) arranged hierarchically. With some visual aids I then progressively abstract and generalize until I reach

2), the visual **model**, this being supported by a mathematical model. I have called these models hierarchical set and element plans and equations. They have a variety of representations and some generalizations which will be presented and discussed.

3) In the **deduction** section I show how the data from different living systems can be encoded into the models. This then describes and orders these systems hierarchically and allows one to develop methods of classifying organisms and predicting data within the life sciences.

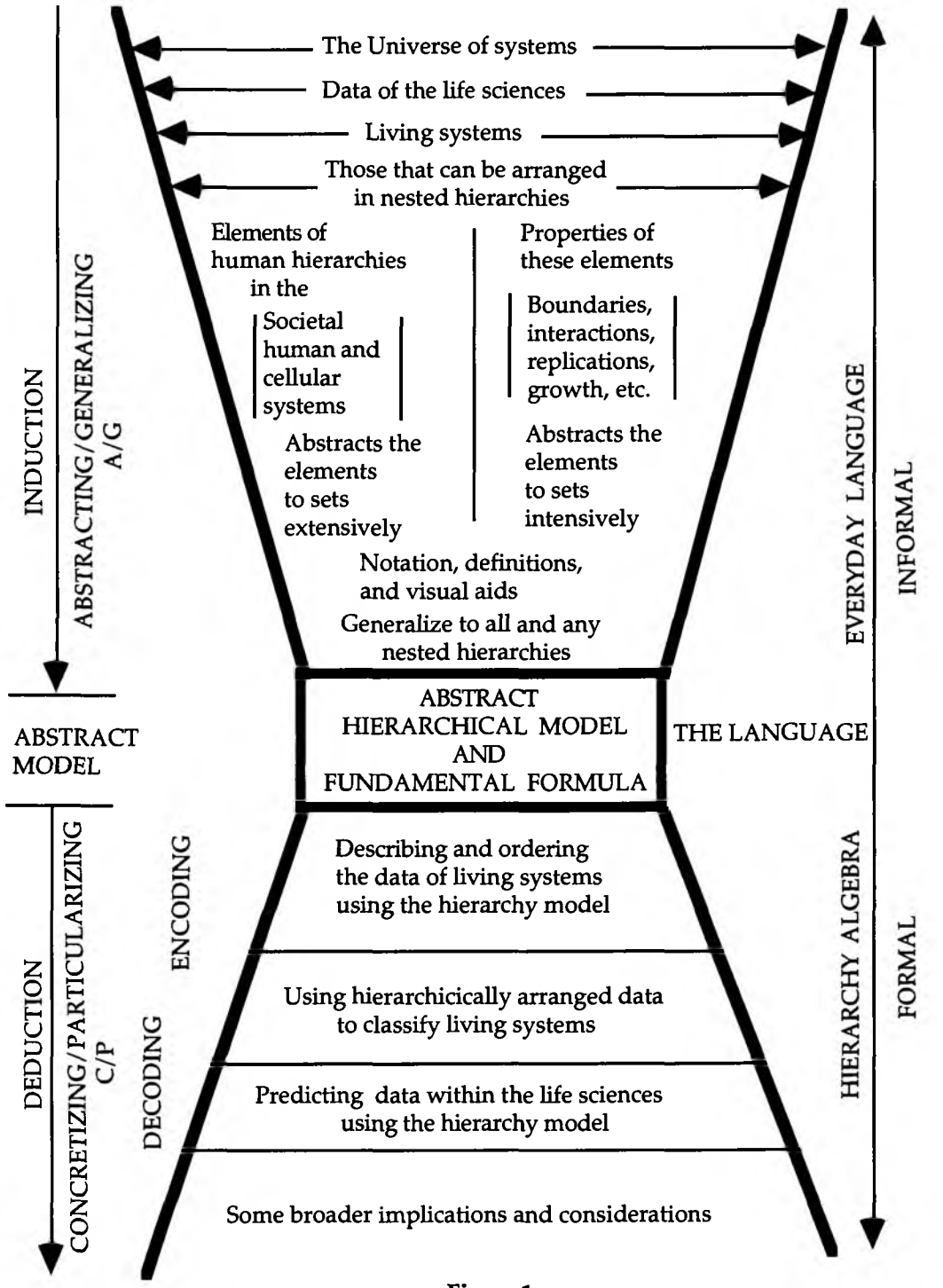


Figure 1.

In more detail, the **induction** process begins with the concrete and, with progressive abstraction and generalization, one arrives at the visual and mathematical models. This inductive

process is shown in the upper half of the hourglass of Figure 1. Because it sometimes is difficult for me to differentiate between abstraction and/or generalization, I will often note this as simply A/G.

Beginning at the top of the scheme, of the universe of systems, we are interested only in living systems (or systems that fall within living systems). Of these, we are interested in only those that have been investigated and for which there is some data. Finally, most if not all of these can be arranged as whole-part or nested hierarchies. I will abstract these out for our attention. Since my area of expertise is humans, for the moment I will concentrate on these. Because I am using set or Boolean algebra as my mathematical instrument, I have divided the data into two classes: those statements that deal with elements (where the elements are living objects such as families, organs, and organelles), and those statements that deal with predicates (properties) of or between these elements.

By a hierarchical system, I mean a system that is a nested or is a whole-part hierarchy, the thing in itself being nested. Another type of hierarchy might be called a control or authoritative hierarchy where the control is nested (as in the Army or the Catholic church). I will not be concerned with this type of hierarchy in this book.

We have left three hierarchical human systems: 1) The societal system (elements being the nation, communities, extended families, nuclear families, and humans), 2) the individual human system (elements being the human, organ systems, organs, tissues, and cells), and 3) the cellular system (elements being the cell, organelles, macromolecules, molecules, and atoms). These objects, wholes, or elements are defined extensively. They are named, pointed to, or enumerated. They then may be generalized to any and all human systems, which, in turn, may be abstracted to any hierarchical subset. This generalized hierarchy contains many individual hierarchies, each with L (L=2,3,...) levels of organization.

Now let us turn to predicates or the properties of these hierarchically arranged elements (systems). In studying them, one comes up with a number of properties that are very important biologically and are also present on most levels of hierarchical organization. Examples of such properties are **boundaries** containing elements, **interactions between elements, interactions between levels, conduits** between levels, **growth** and **replication** of elements, element **self movement, self support,** and so on. We can use these properties of the elements to define subsets **intensively**. By this, I mean that the set is defined by the elements that have a particular property. This is in contrast to the **extensively** defined subsets noted above.

I can further generalize the many properties to any property $P_i(l)$ which can be arranged hierarchically on level l, l = 1,2,...L. and where i = 1,2,...nth property. Some of these properties can be symbolically represented in the floor and elevation plans. By letting $P_1(l) = B(l)$ (any boundary around elements on level l), and $P_2(l) = I(l)$ (any interaction between elements on level l), one locates the elements in hierarchy space. If one then places $P_3(l)$ into the generalized hierarchy of objects, and generalizes to L levels and $P_n(l)$ properties, one arrives at what I have called the hierarchy property model and equation, the fundamental model and formula of this paper. This is the deductive model we have been seeking. The graphics (set and element plans) are isomorphic to the equations. I should reassure the reader that because of the visual models (architectural plans), one does not need to understand the mathematics to understand the ideas or concepts.

The visual **abstract model** has a number of parameters and many variables. The floor and elevation plans are drawn in both a set (intensive) and an element (extensive) form as are their supporting equations. The algebra can be written in algebraic, binary, and a decimal form, and Venn diagrams, truth tables, and lattices will be used. It should be noted that all data in the inductive process is expressed in informal or everyday language.

The deductive system or formal language I shall be using is abstract Boolean algebra. There are a number of "dialects" of this language, called propositional, set, circuit, probability, or machine algebra, depending upon the subject matter to whichthe Boolean algebra is applied. I am calling the dialect developed in this paper **hierarchy algebra**. Each of these algebras has its own peculiarities, and this one does also. If the reader knows none of these languages, he will miss some of the beauty of what is to follow.

Boolean algebra seems most suitable for exploring biological structures.

a) The really important properties (variables) in the biological sciences often do not have values that can be measured with integers or the reals. In set algebra, however, they are easily measured. That is, if P is any biological property of a subset of elements, then in this algebra it has only four possible subsets; i) P, ii) P', iii) $P \cup P'$, and iv) $P \cap P'$. We may sometimes choose to ignore P and can with iii) since $P \cup P' = 1$. With iv) we have $P \cap P' = \emptyset$ or the empty set.

b) In living systems there is tremendous variability in the numbers of elements of concern, both on any given level and between different levels. This variability makes it difficult for any one mathematical system to describe them. Set algebra deals with this by grouping the one or many elements into sets and dealing with these.

c) Boolean algebra also has a long and honorable history in exploring complex structures. As propositional algebra, it has been most successful in investigating the structure of logic and, as set algebra, it has served a similar function in mathematics. It has been called the universal language (Leibniz, Peano, and Boole). As circuit algebra, it plays an important role in computer circuitry and, in its binary form, it serves as the machine language of computers.

In the **deductive** half of Figure 1, I want to first **describe and order** the data of any living system using the hierarchy model (plans and equations). This is done in two steps: 1) Having used the special properties boundary and interaction to define the levels of the hierarchy space model, I now encode this with the data from a specific living system I wish to describe. Besides specifying the number of levels L and some other parameters of the system, one encodes each variable with one of its' four subsets (values), a) $P_i(l) \cup P'_i(l)$, b) $P_i(l)$, c) $P'_i(l)$, and d) $P_i(l) \cap P_i'(l) = \emptyset$, the empty set. The resulting model (subset) relates the elements making up the hierarchy one to another on the same level, as well as to other elements on different levels. We have a horizontal as well as a vertical dimension and for this reason I have called this the hierarchy space model. 2) To the space model one can add other properties of the living system and this I have called the hierarchy property model. If one wishes to look at a particular property $P(l)$ of the above living system hierarchically, one again encodes into the model the values found in the data regarding this property in this system. Later, different properties can be encoded into the model to give us a more complete description. We now have both a visual and algebraically hierarchically ordered description of this living system. We may then go ahead and describe other living systems and can compare one with another. This adds explanatory coherence to the whole deductive system.

Classifying these living systems hierarchically follows readily. Having encoded biological data into the hierarchy model and algebra, one can manipulate it using the rules of set algebra, and we should be able to **predict** data within the life sciences. At the end of the deduction one must then decode the symbols back into biological terms. One interprets the models and equations, either at the set or element level, the latter being the more concrete. I close the deductive section with some inferences on some broader issues.

C) INDUCTION TO THE HIERARCHY MODEL

1) Arranging Biological Systems Hierarchically

We begin the induction by arranging as many living or biological systems as possible into whole-part and nested hierarchies in which the parts (elements) below are contained in the whole (set of these elements) above. In Table I, I have arranged in the second column under biological systems one hierarchical system that contains most of these living systems. In the first column, I have presented a notation scheme in which the hierarchical H(*l*) level of organization is shown. I have chosen as my base reference level, or benchmark H(0), the complex organism or human being. Levels above this (which are outside the organism) are positive, while levels below this (which are inside the organism) are negative. *l* is the number of the particular level with reference to H(0). One can insert extra levels between these (e.g., ecosystems within ecosystems, or supracommunity or supranational systems) for particular biological or sociological systems. One can also remove a level if indicated (e.g., reproductive strategists such as many fish and insects have no H(1) nuclear family). It will also be noted that since complex organisms were labeled H(0), some simpler organisms are given a negative notation. Thus eukariotes are H(-4), and multicellular, multileveled organisms that are made up of 4, 3, and 2 levels are labeled H(-3), H(-2), and H(-1).

Abstract	Biological systems	Human systems	A/S	Science
H(5)	World	World	↑	
H(4)	Biogeographic regional	Nation	↑	
H(3)	Ecosystems	Communities	↑	Ecology or
H(2)	Populations	Extended families		Sociology
H(1)	Families	Nuclear families	A S	
H(0)	Complex organisms	Humans	n y	
H(-1)	Simpler organisms which	Organ systems	a n	
H(-2)	are multileveled and	Organs	l t	
H(-3)	multicellular	Tissues	y h	Biology
H(-4)	Eukariotes-cells	Cells	s e	
H(-5)	Prokariotes-Organelles	Organelles	i s	
H(-6)	Macromolecules	Macromolecules	s i	
H(-7)	Molecules	Molecules	s	Chemistry
H(-8)	Atoms	Atoms	↓	
H(-9)	Nucleons	Nucleons	↓	Physics
H(-10)	Quarks	Quarks	↓	

Table I.

In the third column, I have shown a particular hierarchy or system, the human system. The same general statements made about biological systems can also be made about this one. This is the one that I am most familiar with and the one I will use in the induction of the hierarchy model.

In the fourth column, I have shown the **analysis** of the whole into parts, moving downward, and the **synthesis** of parts into wholes, moving upwards. I will be using these words in this sense in this book.

Finally, in the fifth column, I have grouped levels together under the science that studies them. Thus, levels H(0) through H(5) are studied as ecology in biological systems and as sociology in human systems. Levels H(0) down to H(-6) are studied in biology, and levels H(-6) through H(-8) in chemistry. Levels H(-8) through H(-10) are studied in physics. In addition, certain aspects of all levels may be described using the terms of physics (e.g., distance, time, velocity, temperature).

2) Rearranging the Human Hierarchy

In order to proceed with my research on hierarchical systems, I must simplify, and I have done this in Table II below. I have left out the levels at the extremes H(-9), H(-10), and H(5). I have also grouped together the remaining into three five-level systems called the social system, the human system, and the cellular system. I have changed he notation so that instead of a fixed benchmark H(0), we have a floating or changeable benchmark H(1), H(1) being interpreted differently in different systems, as shown below.

<u>The Social system</u>	
Nation	H(5)
Communities	H(4)
Extended families	H(3)
Nuclear families	H(2)
Humans	H(1)
<u>The Human system</u>	
Human	H(5)
Organ systems	H(4)
Organs	H(3)
Tissues	H(2)
Cells	H(1)
<u>The Cellular system.</u>	
Cell	H(5)
Organelles	H(4)
Macromolecules	H(3)
Molecules	H(2)
Atoms	H(1)

Table II.

3) Generalizing the Human Hierarchy

Table II can be further generalized to any five-level living system, as shown in Table III. Here the primitive H(1) is a sub-sub-sub-subsystem. I have also differentiated between elements and subsets. In the column under element, I am considering only one element on each level, for instance, a particular organ or particular molecule contained in that system. This is shown more clearly using the abstract notation in the second column on the left.

An element		Subsets	
A living system contains	H(5)	A living system & all its	H(5)
a sub-system, contains	H(4;i)	sub-systems and their	H(4)
a sub-sub-system, contains	H(3;i,j)	sub-sub-systems and their	H(3)
a sub-sub-sub-system etc.	H(2;i,j,k)	sub-sub-sub-systems and	H(2)
a sub-sub-sub-sub-system.	H(1;i,j,k,l)	sub-sub-sub-sub-systems	H(1)

$i=1,2,...,n(4)$, $j=1,2,...,n(3;i)$, $k=1,2,...,n(2;i,j)$, $l=1,2,...,n(1;i,j,k)$

Table III.

For example, in H(4;i) ,the 4 indicates that the element is on the fourth level of the hierarchy, while the the i following the semicolon indicates that it is the ith element on that level. If we consider all of the elements on a given level, we get a subset of elements on that level, this being shown in the third and fourth columns. Thus, by letting i = 1,2,..,n(4), we have labeled all of the elements on the fourth level, which defines the subset H(4). This allows us to go from elements to subsets, as shown at the bottom of Table III.

In summary, we have progressed from all the living systems that can be placed into a generalized whole-part nested hierarchy to a special instance, the human hierarchy. The latter has been split up into three local hierarchies, which in turn were generalized to one five-level abstract hierarchy or system. The notation system was enlarged so as to include both elements and subsets. From this one can proceed further with the work of abstracting the elements.

4) Visual Models or Architectural Renditions

a) Elevation plans and some definitions

Figure 2 shows a visual model or architectural rendition of what I have called elevation plans. I will use this figure to define a number of terms that will be used throughout the book. On the left (above A) I have shown an element elevation plan on which one element from each level is shown, similar to the element column of Table 3. The lower elements are contained by the upper ones.

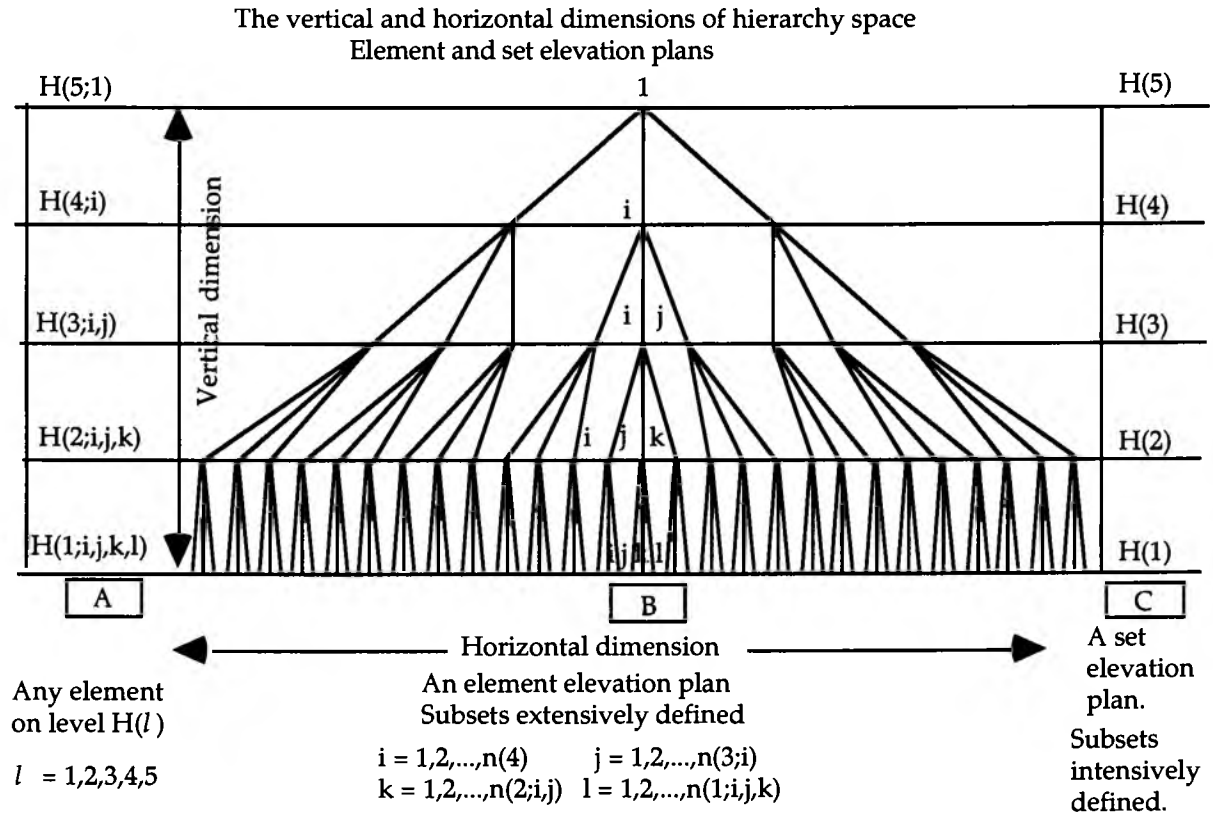


Figure 2.

In the middle (above B), I have shown another elevation plan in which the elements on a given level are shown. On the right (above C), I have shown the subset notation standing for all of the elements

on that level. For example, H(4) = {H(4;i) | i = 2,3,...,n(4)}. This is similar to the subset column of Table III. **Level** stands for the level of organization.

Sets may be defined in two ways, extensively and intensively. In the **extensive definition** the elements of the set are labeled, pointed to, or enumerated, while in the **intensive definition** all elements or members of the set have the same property. Thus the subsets of C are are intensively defined, each element having the property of being on a particular level. I have called this a **set elevation plan**. On the other hand, the subsets of B are defined both intensively (by level) and extensively (by enumeration). I have called this an **element elevation plan**. Elevation plans show the **vertical dimension** most clearly while Figure 2B shows the **horizontal dimension** most clearly. The two dimensions, vertical and horizontal, define what I have called **hierarchy space**. Living systems occupy this space.

b) Floor plans

Another visual model, or architectural rendition, of hierarchical systems is the element and set floor plans as shown in Figure 3.

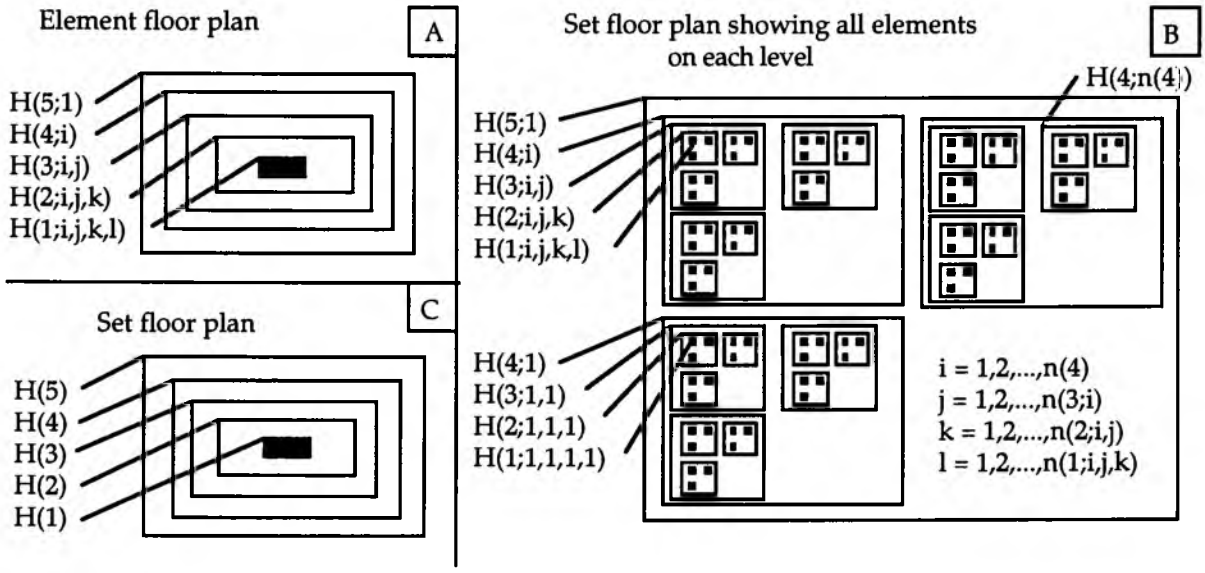


Figure 3.

In Figure 3A, one sees an element floor plan, one element being on each floor, beginning with H(1;i,j,k,l) on the ground floor and ascending until we get to H(5) on the top floor (a singleton). Figure 3B shows an element floor plan defined intensively and extensively with some of the elements on a given floor enumerated. Figure 3C shows the set floor plan defined intensively, each "box" representing all of the elements on that level. A,B, and C of Figure 2 and A,B, and C of Figure 3 are different views of the same things.

5) In Perspective, the Vertical and Horizontal Dimensions and Hierarchy Space

The question arises, how do these two dimensions and this space relate to the traditional three dimensional space? In Figure 4 ,I hope to clarify this. Consider Figures 2A and 3A, element elevation and floor plans. If one combines these he obtains the perspective view shown in Figure 4A. The three traditional dimensions (X, Y, Z coordinates) are seen to be located on each level

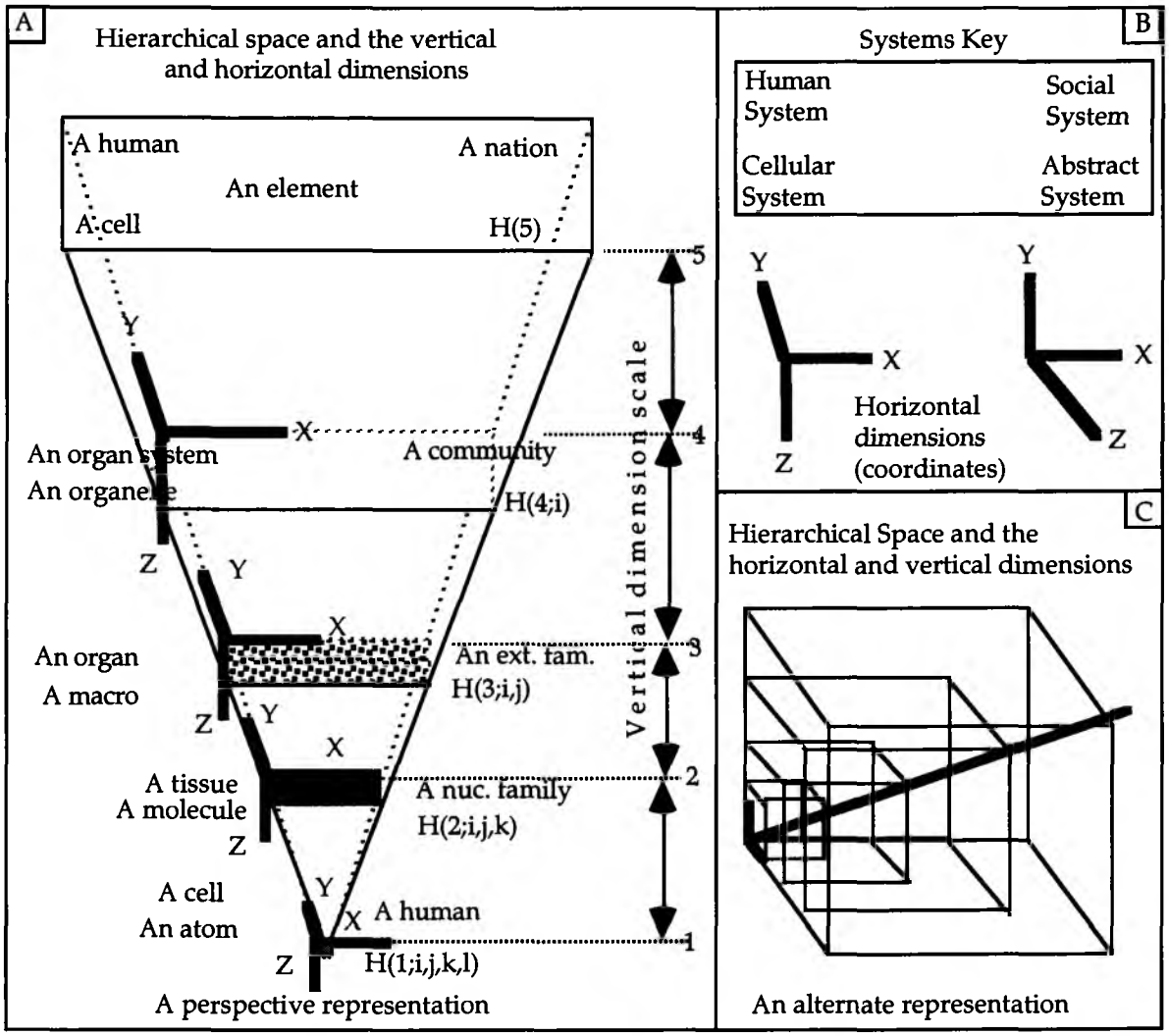


Figure 4.

(floor). These are what I have called the horizontal dimension. What is different about them is their scale. At the bottom the scale is very small, but it gets larger as one ascends. This is the vertical dimension. It might be called a **scaling dimension**. To get a better feel for the scaling, I have interpreted the **abstract** notation in Figure 4A, using the key of Figure 4B. Thus I have shown the vertical dimension of the **societal, human, cellular, and abstract** hierarchies (as in Table II). One uses a different scale for measuring distances at the nation level than one uses at the human or community level. The same holds true in the other systems. To make this even more concrete, I have shown in Figure 5 some typical scales for all the systems considered above.

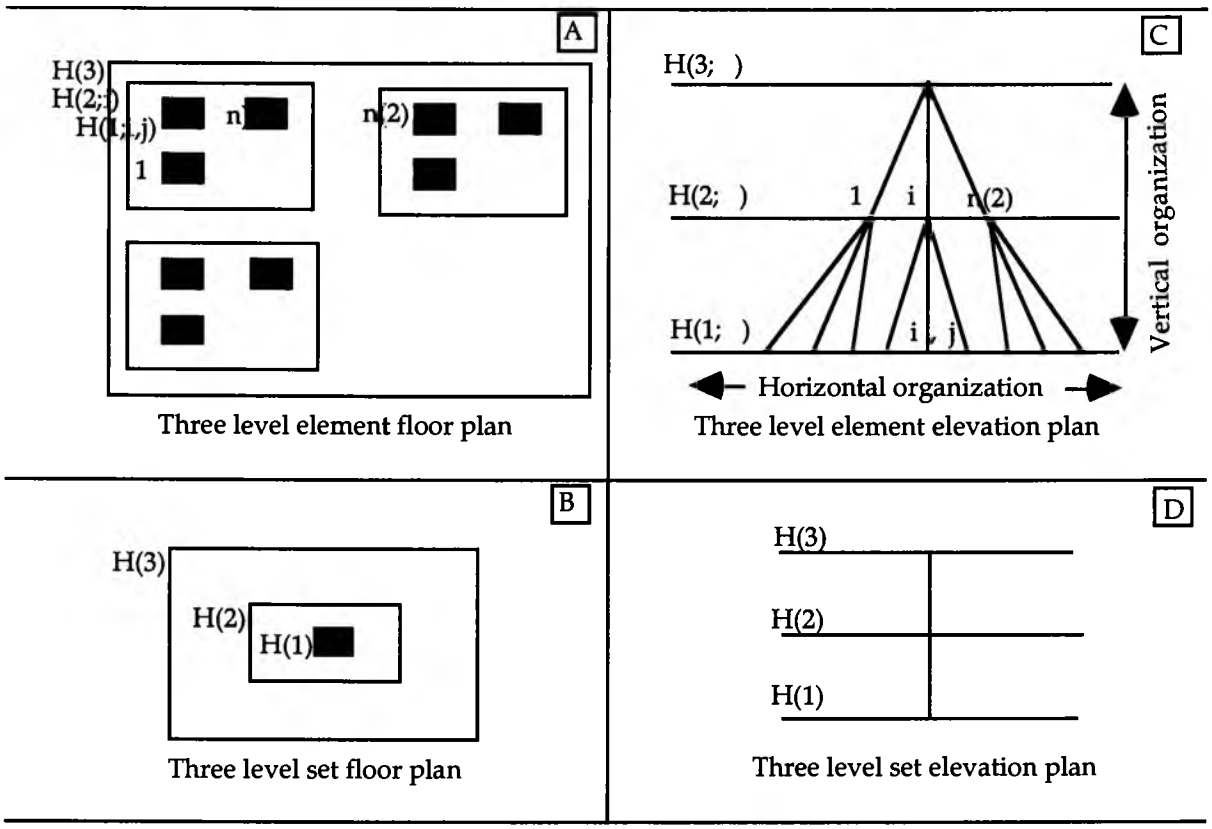
Human Space			Social Space		
Level	Scale	Svstem	Level	Scale	Svstem
5	10^{0} = 1 meter	= human	5	10^{5} = 100 kilom.	= nation
4	10^{-1} = 10 cm.	= organ system	4	10^{3} = 1 kilom.	= community
3	10^{-2} = 1 cm.	= organ	3	10^{2} = 100 m.	= ext. family
2	10^{-3} = 1 mm.	= tissue	2	10^{1} = 10 m.	= nuc. family
1	10^{-5} = 10 micr.	= cell	1	10^{0} = 1 m.	= human
Cellular Space			Abstract Space		
Level	Scale	System	Level	Scale	System
5	10^{-5} = 10 micr.	= cell	5	10^{5} = units	= system
4	10^{-6} = 1 micron	= organelle	4	10^{4} = units	= sub system
3	10^{-7} = .1 micron	= macromolecule	3	10^{3} = units	= sub-s-s
2	10^{-9} = 10 angstroms	= molecule	2	10^{2} = units	= sub-s-s-s
1	10^{-10}= 1 angstrom	= atom	1	10^{1} = units	= sub-s-s-s-s

Figure 5.

For each system I have shown the level, the scale in powers of 10 meters, and the subsystem on that level. The total span of the scales is great, going from say 10^{5} meters for a nation down to 10^{-10} meters for the atom. An excellent presentation of the vertical dimension and this scaling is found in Eimes and Morrison's book *Powers of Ten* , from which I obtained the scaling numbers of Figure 5. Returning to Figure 4, this time 4C, I have shown an alternate and more compact representation of hierarchy space and the vertical and horizontal dimensions. Again the X, Y, and Z coordinates are shown, but this time the origin is shifted to a new position with each higher level and higher magnification.

6) Describing and Ordering Living Systems Hierarchically. From Abstract to Concrete. Two Simple Examples of Encoding and Decoding

At this point, it might be worthwhile to skip ahead a bit, both to see where we are going and also to get more concrete, since specific examples are easier to understand than the abstract presentation. I will give two examples of interpreting the abstract. I have shown, in Figure 6 below, the abstract three-level hierarchy with both the element and set elevation and floor plans. I would like to interpret this in two ways. First, I will encode it for a hierarchically ordered description of an extended family and then encode it for a description of the heart. By encoding I mean to enter data regarding the particular living system into the model using the notation system of the model. This involves assigning a hierarchy symbol (or number) to each element of the living system under consideration, in this case first to the extended family and second to the heart. To decode, one reverses the process.



$$\{H(3)\} = \{H(2;i) \mid i = 1,2,...,n(2)\} \cap \{H(2;i)\}$$
$$\{H(2;i)\} = \{H(1;i,j) \mid j = 1,2,...,n(1;i)\}$$

Figure 6.

Consider the Grant family and their hierarchical representation.

The third level----The extended Samuel B. Grant family = H(3)

The second level---His nuclear family members are H(2;i) | i=1,2,...,n(2). Let the element i = 1,2,3,4,5 and assign to nuclear family members.

Samuel B. = H(2;1)
Natalie N. = H(2;2)
John M. = H(2;3)
Neville = H(2;4)
Samuel B. Jr. = H(2;5)

The 1st level---The individual extended family members are H(1;i,j) | i = 1,2,...,n(2) and j = 1,2,...,n(1;i). Let i = 1,2,3,4,5, as before and j = 1,2,3,4,5 and assign to individual family members.

Samuel B. = H(1;1,1)
Natalie N. = H(1;2,1)
Neville = H(1;4,1)
Diane C. = H(1;4,2)
Johanna = H(1;4,3)
Bevin = H(1;4,4)
Natasha = H(1;4,5)
John M. = H(1;3,1)
Margaret T. = H(1;3,2)
Natalie T. = H(1;3,3)
Samuel B. Jr. = H(1;5,1)
Patricia D. = H(1;5,2)
Christopher N. = H(1;5,3)
David D. = H(1;5,4)

----------Encoding---------->
<----------Decoding----------

In Figure 7, I have encoded the element elevation plan with the various members of the Grant family, giving all of them a number and thus ordering and describing them hierarchically. They may be decoded (interpreted) by substituting the name for the number.

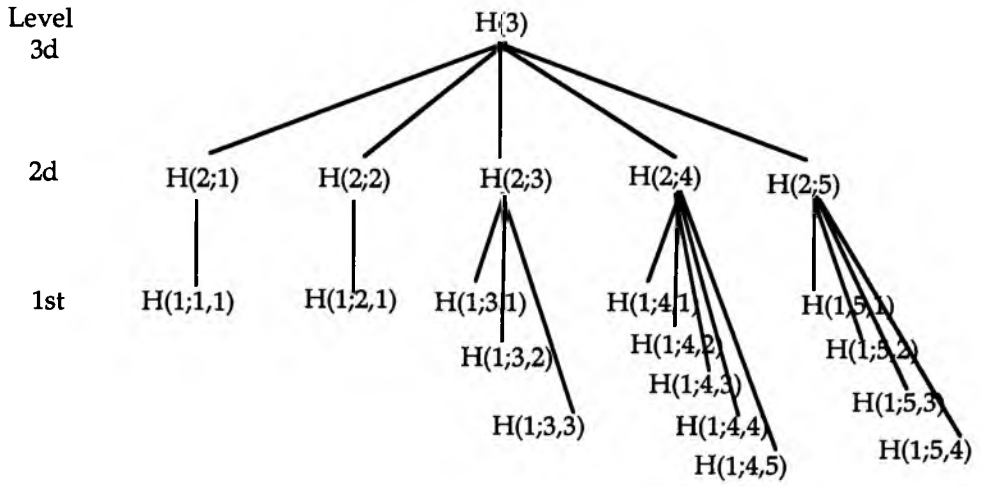


Figure 7.

In Figure 8, I have encoded the model in a different way. Consider the human system, and more particularly the heart. In the figure, I have described its three levels, that is, the organ, the tissue, and the cellular levels. I have encoded (enumerated) the three tissues involved. Since I don't know the number of cells in each tissue, I have let them number n(1;1), n(1;2) and n(1;3).

Regarding the nomenclature of the elements again, the first number in the parentheses following H(*l*) represents the level, a subset. For the top level this is H(3). The first index after the semicolon, i, is an element H(2;i) on the second level and is the nuclear family (or tissue).

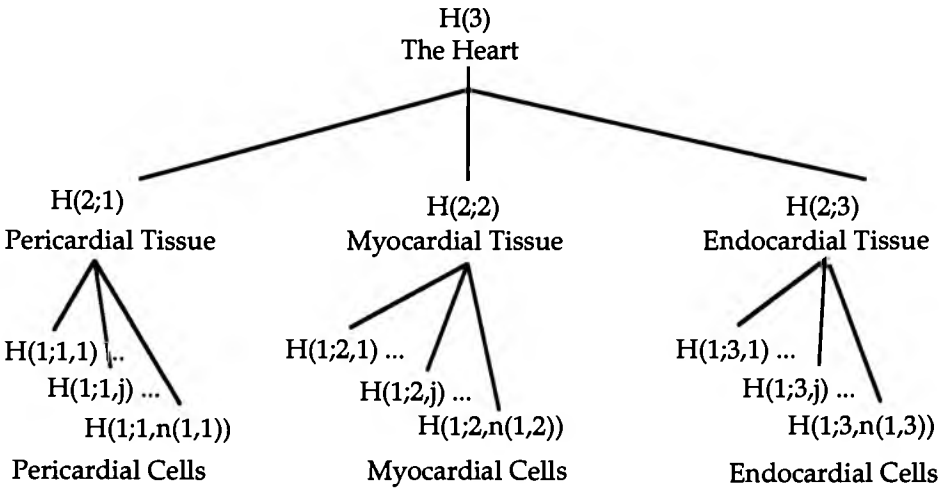
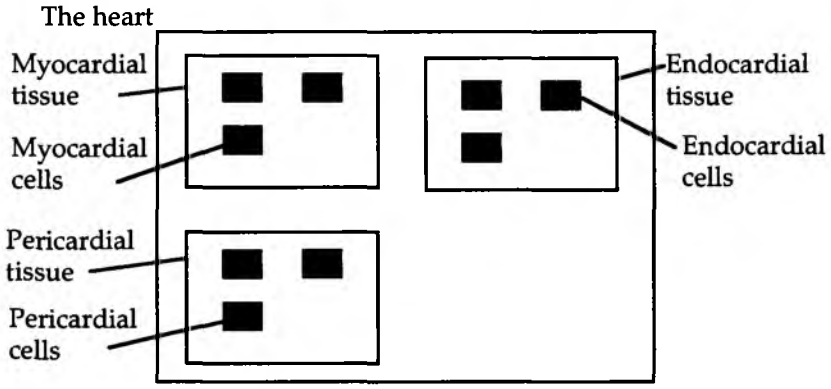


Figure 8.

The next index j, is an element H(1;i,j) on the third level and is an individual (or cell). Once again, it will be noted I have not enumerated the subset H(3) in either the extended family or the heart in order to cut down on the number of indices. Unless otherwise stated, the top subset has only one element: it is always a singleton.

The set and **element floor plan** of Figure 6 can also be decoded and, in Figure 9, I have shown the latter decoded to the hierarchical structure of the heart.



A decoding of H(3)

Figure 9.

By way of these examples, I hope to have shown that the data regarding most living systems can be organized hierarchically. One would hope that the resulting paintings would be Rembrandtian. Instead, we end up with stick figures. However, the paint, brush, and canvas have been chosen, and the rest of the book will be edging towards that ideal.

These examples show in microcosm what is being presented in this book. I have developed a notation system within which one can place data regarding the hierarchical structure of living systems. One of the nice things about this system is that it can be manipulated using the rules of finite set theory and algebra. How useful this is has yet to be demonstrated.

Thus far I have described the general method, abstracted the elements from living hierarchical systems to sets, introduced levels of organization, defined the set H(l) using the elements H(l;i) | i=1,2,...,n(l), and the set H(l+1) above in terms of the subsets{H(l;i) | i=1,2,...,n(l) } = H(l) below. But I do not have a set algebra. Also, I have said nothing about how the levels of organization are defined. We have seen the many below become the one above. What causes this to occur? In the next section I will turn to these problems and questions.

7) Some Important Biological Properties. Working Towards the Model

In symbolic logic one has the subject of concern and then one has the predicates or properties of this subject. The subject, elements or living objects, I have written of in the previous sections. In the present section I will concentrate on properties. **I am interested in those properties that occur on many levels of organization.** These have been a central part of James Grier Miller's encyclopedic work on hierarchical *Living Systems.* Two are of particular importance, for it is they that determine how the many elements (the parts) below become a whole new element above. It is these that define the levels of organization. In deciding what properties define hierarchical levels in living systems, one must ask oneself what properties keep elements (parts) together so that they can be said to form a new element at a higher level (a whole). If one refers to Table I and has some knowledge of these biological systems, I believe it will be seen that there are two properties that fulfill this function. But first, a word on the set notation used.

THE SET NOTATION USED

(1) Sets are indicated by capital letters, thus A; sometimes for clarity by its elements inbrackets, thus {A}.

(2) Elements or members of A are indicated by indices or numbers following a semicolon, as in A;i, where the element i = 1,2,...,n. This is often written A;i | i=1,2,...,n.

(3) ∩ is the intersection or logical product, sometimes called simply product. Sometimes ∩ may be left out. Thus A ∩ B may be written AB. [Editor's note: The intersection of two sets consists of the elements they have in common. It is sometimes helpful to read "∩" as "and."]

(4) ∪ is the union or logical sum, sometimes called sum. The union of A and B would be A ∪ B. [Editor's note: The union of two sets consists of the elements in either set. It is sometimes helpful to read "∪" as "or."]

(5) A' is the complement of A. [Editor's note: The complement of a set consists of the elements not in that set. It is sometimes helpful to read "A'" as "not A."]

(6) Ø is the empty set.

(7) I and sometimes 1 represent the universe of subsets.

(8) ∪ A(i) (where i = 1,2,...,n) = A(1) ∪ A(2) ∪ ... ∪ A(n).

(9) ∩ A(i) (where i = 1,2,...,n) = A(1) ∩ A(2) ∩ ... ∩ A(n).

(10) A ⊃ B means that A is contained in, or is a subset of B. [Editor's note: In this case, each element of A is also an element of B.]

(11) [Editor's note: B ⊃ A means that B contains A. Again, this means that each element of A is also an element of B.]

(1) The Boundary Property (Variable) on Level *l*, B(*l*) ∪ B'(*l*).

If one consults Table I, he will note that, at most levels, we have a boundary that surrounds or contains the elements together so that the many parts become one whole at the next higher level (see Appendix A). Thus the cell has a cell membrane which contains many organelles, an organ may have an epithelium which contains the tissues, a human has a skin which surrounds organ systems, the family has a house, and so on. The first property that makes many into one in our model is therefore the boundary property B(*l*), *l* representing the boundary on the *l*th level. It will be noted that this boundary is "semi-permeable." That is, it will contain certain classes of elements but not others. Or, it may permit passage of certain elements at one time and not permit passage at another time. It seems to have entrances and exits that may be open or closed. I will come back to this later in the book. Inspecting again Table I, one finds that while some elements may be contained or surrounded by a boundary, others are not. Thus an organelle membrane may contain some macromolecules of the cell; other macromolecules are outside of organelles and not contained by organelle membranes. Some tissues are contained by an organ epithelium, while others are not. In a community, some extended families may live in a "compound" while others may not. Thus B(*l*) has a complement B'(*l*). As shown in Figure 10A, B(*l*) is symbolized as a box while B'(*l*) is a dotted box. If H(1) is a set of persons living

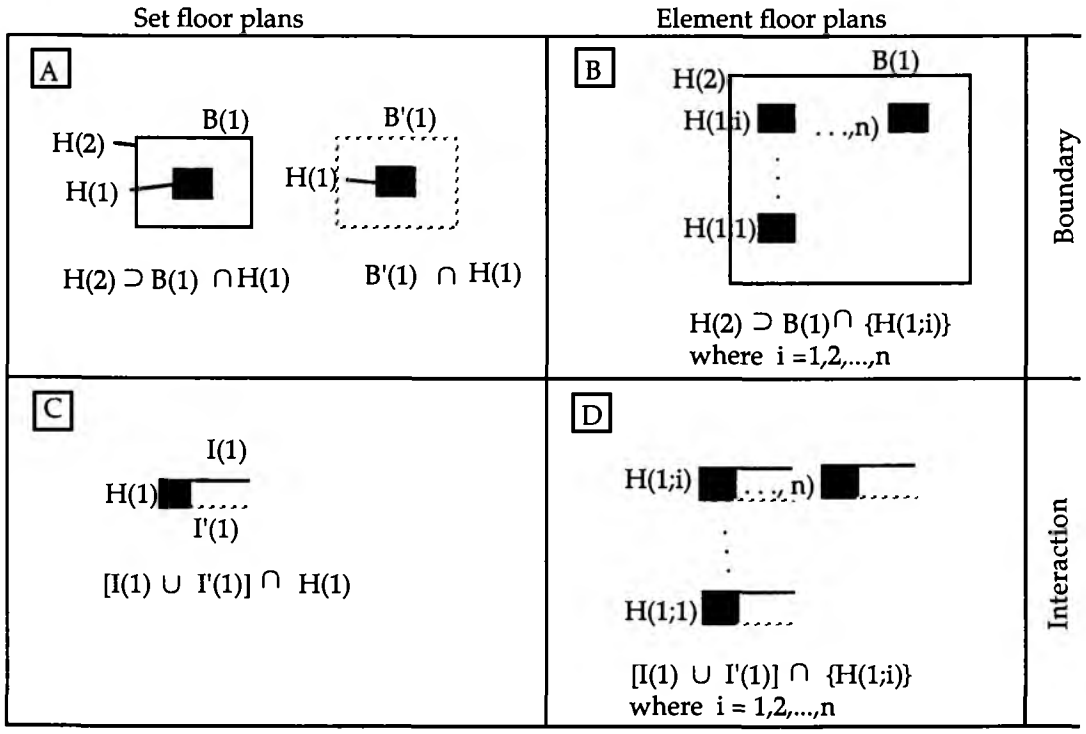


Figure 10.

in a community and B(1) is a house, then B(1) ∩ H(1) is the subset of persons living in a home and B'(1) ∩ H(1) is the subset that have no home. It will be noted that B(1) ∩ H(1) raises the level to H(2) while B'(1) ∩ H(1) does not raise the level. That is, B'(1) does not collect together any elements. We can say that H(2) ⊃ B(1) ∩ H(1). I would also like to reinforce the notation for elements as shown in Figure 10B. If H(1) is a set of elements, then H(1;i) is any element in this set and H(1) = {H(1;i)}, where i = 1,2,...,n. This often will be written simply as H(1) = {H(1;i) | i = 1,2,...,n}.

(2) The Interactional Property (Variable) on Level *l*, I(*l*) ∪ I'(*l*).

If once again one peruses Table I, it will be seen that there are some instances when a boundary does not make the many become one. Thus, we have no boundary making atoms a molecule, and an extended family may not be gathered together in a family compound. And yet both the molecule and the extended family can become a whole at a higher level. It is here that we introduce our second level defining property, interactions or bonds between elements. It is chemical bonds between atoms that make many atoms into one molecule, and extended family bonds that allow one to refer to the extended family. Interactions or bonds between elements seem to be present at almost every level in Table I (also see Appendix A). Thus, nucleons bond or interact with each other to form atoms, organs interact with each other in forming organ systems, and humans interact or bond in forming the nuclear family etc. I will abbreviate this interaction as I(*l*) where *l* again stands for the level we are refering to. If H(1) is the set of 1st level atoms in a given organelle, then I(1) ∩ H(1) is a subset of these atoms that interact with other atoms. In other words, they are the molecules of this organelle. Then we can let the complement of I(1), I'(1) ∩ H(1) be the subset of atoms in the organelle that are not interacting with one another, the ions in this organelle. To use another example, in a household, I(1) ∩ H(1) is the set of those persons that share the family bond (mother, father, son, etc.), while I'(1) ∩ H(1) consists of those that don't (maid, guest, baby sitter, etc.). Those elements that interact, I(1)H(1), form a new whole that we call H(2;1). Thus we write H(2;1) or simply H(2) ⊃ I(1)H(1) for the nuclear family.

In the lower half of Figure 10 above, I have illustrated I(1) and I'(1) in set and element floor plans. It is seen that in the set plan 10C, I(1) is represented by a solid line coming from the set square H(1), while, in the element plan, 10D, it comes from the element squares, H(1;1), H(1;2),...H(1;n). I'(1), or no interaction, is represented by a dotted line. It should be reemphasized that, in this model, I(*l*) is a two-valued variable and not one measured by the reals. That is, it is either present, or not present. It is not multivalued like length or mass. **It is this property of set algebra that makes biology easier to approach mathematically. Many important biological properties have two easily measurable values but not multiple measurable ones.**

Several other points should be made about interactions. First, they are transmitted by something. Thus gluons transmit the strong interaction, photons transmit the chemical bond, molecules transmit intercellular interactions, and multiple agents transmit interpersonal family interactions. Following the lead of physicists I will call these transmitting agents interactons. Thus, we may have one or multiple interactons accounting for the interaction between elements at any one given level. Secondly, as emphasized by Simon, the interactions between elements at any one level have a certain response frequency or period. In general, the lower down on the hierarchy (the smaller the elements), the shorter the interactional period and more rapid the frequency of interaction. This has been used by him and others to define levels of organization. I find it easier, however, to think in terms of particular interactions than in terms of response frequency rates.

Manipulating the notation to get a model and algebra

I believe we have now developed the notation and concepts enough that we can begin to manipulate the notation system using the rules of set algebra. The two properties B(*l*) ∪ B'(*l*) and I(*l*) ∪ I'(*l*) can be combined in four different ways, which are the subsets shown in Figure 11 below. A) In the upper left, we have H(2) ⊃ B(1)I(1)H(1), the level being raised from H(1) to H(2) because we have both a boundary and interactions pulling the elements together. **The equation for H(2) can be arrived at from the model by starting at the outside of the figure and working in to the center, picking up properties and/or their complements as we move to H(1).** B) In the lower left, we also raise the level to H(2) because we have a boundary pulling elements together, thus H(2) ⊃ B(1)I'(1)H(1). In practice, this is rarely, if ever, found by itself in a living system. It is frequently

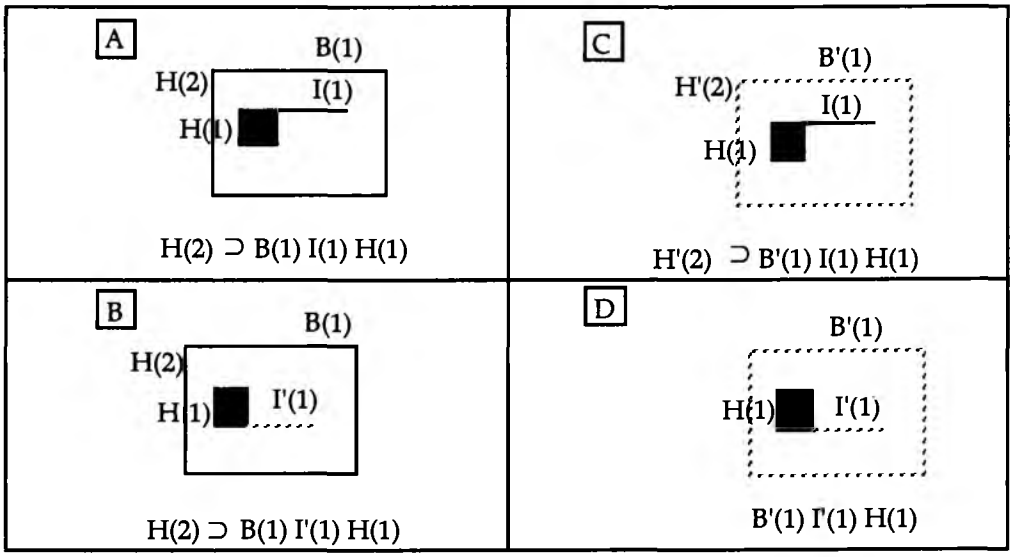


Figure 11.

found in conjunction with B(1) ∩ [I(1) ∪ I'(1)] ∩ H(1) as in Figure 12 below, for instance, ions and molecules in an organelle. C) In the upper right, we have an idiosyncratic notation. When all the elements are held together with interactions, we raise the level (atoms interacting to become a molecule), and I want to indicate this. But I want to indicate that it is of different type from the proceeding H(2). Thus I will say **H'(2) > B'(1)I(1)H(1). Whenever we have a B'(*l*)I(*l*)H(*l*) raising the level, I will define the upper level as H'(*l*+1).** D) In the right lower part of the figure, we have B'(1)I'(1)H(1). Since we have neither a boundary nor interactions to hold the elements together, we have neither H(2) nor H'(2) and the level is not raised, nor do these parts make a whole.

Usually both boundary and interaction properties define a level change although occasionally only one will. The change from an atomic level to a molecular level is defined only by the interactional property. The same is true of some extended families, those without a family compound. B(1)I'(1)H(1) makes elements into a whole H(2) at a higher level for only very brief periods of time. If observed long enough, every element in living systems will eventually interact with some similar element.

With two variables B(1) ∪ B'(1) and I(1) ∪ I'(1) we have 2^2 or 4 products (as above). We also have 2^4 or 16 subsets in the universe of subsets. Exactly which of these 16 subsets we are referring to can be denotated as H(2,S) where S = 0,1,2,...,15. Later we will be solving equations to find out what S is equal to. Until then, we will use the notation H(2) ⊃ (contains) at least one of these products. Note that in H(2,S) a comma (,) separates the level from a subset of that level while in H(2;i), a semicolon (;) separates the level from an element on that level. We may have both: H(2,S;i).

We can add the various products and set plans shown above as one does in any set algebra. For instance, the set plans of Figures 11A) and 11B) sum to form the set plan as shown in Figure 12,1 below. The same is true of subsets and subplans 11A) and 11C) shown in Figure 12-2. Again in Figure 12-2, one can derive the equation from the set plan by beginning at the outside and moving in, this time picking up B(1) ∪ B'(1). In figure 12-3, the four products of Figure 11 can be summed to get the set plan and equation. A) ∪ B) ∪ C) ∪ D) is the complete sum of products. The set plan is a generating set plan.

If one starts on the outside and moves in to H(1), picking up one variable or the other along the way, one obtains a subset plan. If then this is repeated in all possible ways, one obtains the sum of products (4 subplans). Since the universe of subsets can be obtained from this, I will henceforth call this H(2,**I**) where **I** = the universe of 16 subsets or subplans. The subplans, like the algebra, can be simplified. Thus in Figure 12-4 , I have simplified both the algebraic expression and the subset plan using the law of complements.

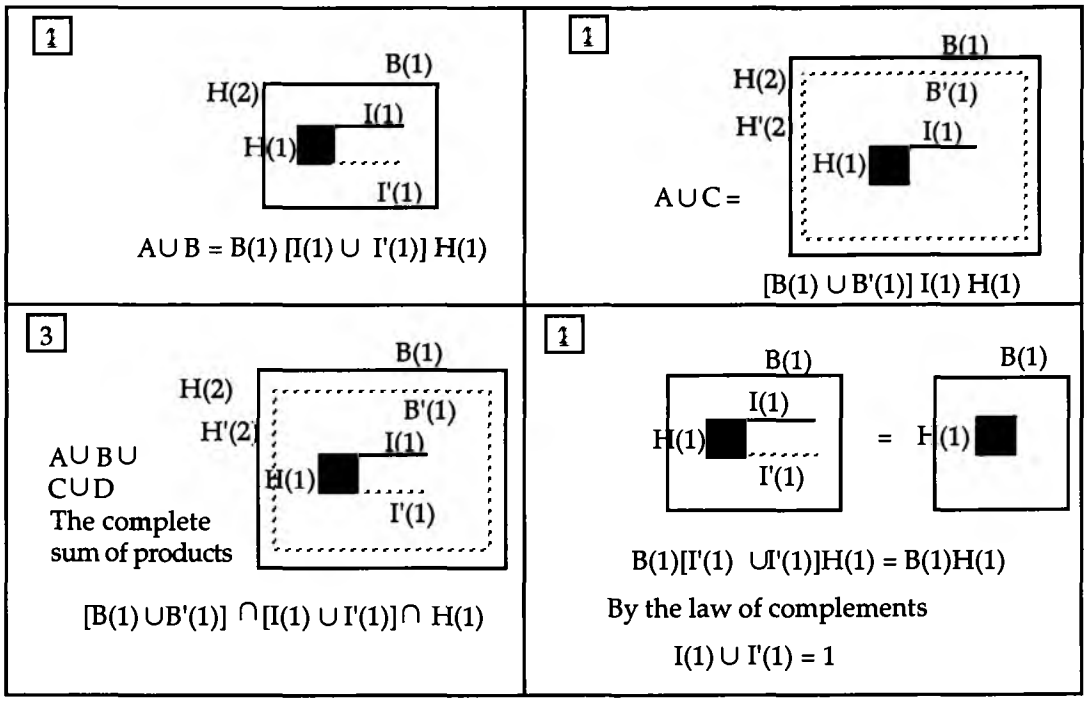
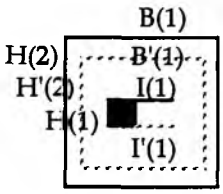


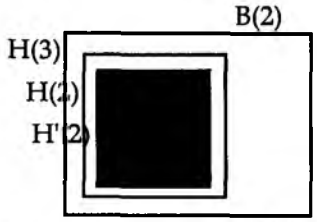
Figure 12.

In the above equations it will be noted that H(2) ∪ H'(2) is the universe of subsets defined by the two variables B(1) and I(1). That is, it contains all of the sixteen subsets pointed out previously. If we want to indicate a particular subset of these, we notate this H(2,S), S = 1, 2, ...,15.

Up to this point we have had an unchanging primitive subject, namely H(1). Now, however, I would like to move to a three-level hierarchy and, to do this, we will have to change the primitive subject to H(2) on the second level. Ordinarily one does not change subjects midstream in set algebra, but **it is this maneuver that permits us to build hierarchy algebra**. Perhaps it is more easily understood by looking at the three-level set plan and its two equations in Figure 13. At the top, we have the set plan for H(2) ∪ H'(2) with the primitive subject **H(1)** (the "black box" = without structure) and its boundaries and interactions. It is described by equation 1). Next, we have the set plan H(3) with the new subject ("black box") **H(2)** ∪ **H'(2)** and its boundaries and interactions. It is described by equation 2). Finally, at the bottom, we have substituted 1) into 2) (set plan and equation) to get the three-level set plan and equation in terms of the primitive **H(1)** and all the boundaries and interactions. Thus, we can describe our set plans using either two equations with a new primitive subject for each level, or with just one equation with but one primitive subject for the entire hierarchy. Each choice has advantages and disadvantages.

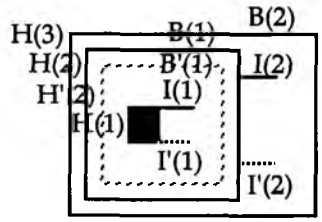


1) $H(2) \cup H'(2) \supset [B(1) \cup B'(1)] \cap [I(1) \cup I'(1)] \cap \mathbf{H(1)}$



2) $H(3) \supset [B(2) \quad] \cap [I(2) \cup I'(2)] \cap \mathbf{[H(2) \cup H'(2)]}$

Substituting 1) into 2), we have



$H(3) \supset [B(2) \quad] \cap I(2) \cup I'(2)] \cap$
$[B(1) \cup B'(1)] \cap [I(1) \cup I'(1)] \cap \mathbf{H(1)}$

Figure 13.

Some other important biological properties found hierarchically in living systems

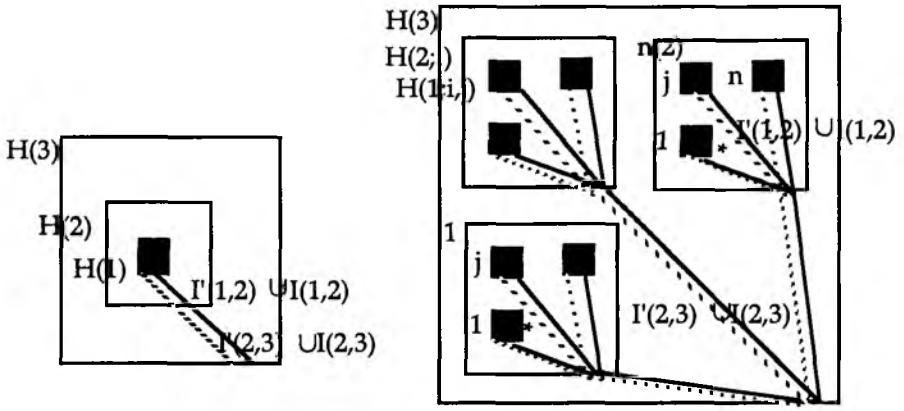
In the last section, I used the two properties, boundary and interaction, to define the levels of hierarchical systems. These independent variables located objects on various levels and defined the hierarchical space of the living system. At the very best, however, this is an anemic description of a living system. I would like to be able to "flesh out" these descriptions, and thus, I must consider other properties that are typical of living systems. These are properties that occur (or not) on a number of different levels, often on most of the levels of Table I. Each of these properties (independent variables) has two values, and thus there are subsets of the property P(l). They are, 1) the property P(l), 2) its complement P'(l), 3) the property and/or complement $P(l) \cup P'(l)$ when each is present on level l. When we want to ignore this property on this level, then $P(l) \cup P'(l) = 1$. 4) The empty subset $P(l) \cap P'(l) = \emptyset$. Usually, I will just consider one or two properties at a time, ignoring the others. Considering more than this at one time, while giving a more comprehensive description, leads to complicated mathematics. I will first describe a number of these properties and the procedure to be followed. Then, in a later section, I will use some of them to look at some living systems vertically.

(3) Vertical Interactions and/or their Absence: I(1,2) ∪ I'(1,2), or More Generally, I(l,l+1) ∪ I'(l,l+1)

In the previous model the interaction property I(l) was between any two or more elements on the same level. It was a horizontal interaction between elements on the same level. However, no hierarchical system could "live" if elements **between** levels were not in some way interactive or interconnected. Thus it is only logical to assume that there are also vertical interactions. These might be called interlevel interactions, while the former are horizontal or intralevel interactions. I will label these vertical interactions I(l,l+1). The absence of such an interaction will be labeled I'(l,l+1). The label obviously is meant to refer to an interaction (or no) between elements on the lth and (l+1)th levels, etc. Once again, these interactions are transmitted by something that I will call a vertical interacton, and we can safely say that what is transmitted is matter-energy and/or information in some sort of package or wave (e.g. nerve impulse). If one studies Table I, I am sure that numerous examples will come to mind (and see Appendix A).

In Figure 14 ,I have shown a set plan on the left and an element plan on the right together with the symbolism I will be using. The solid line connecting, say, levels 1 and 2 is the I(1,2) vertical interaction, while the dotted line to its left is the absence of this interaction, I'(1,2).

To give an example, one might consider two persons (on first level and marked by an asterisk *) that are cousins by the fact that their fathers (on the second level) are brothers. The interactions are familial.



Three level set plan

Three level element plan

Figure 14.

(4) Vertical Conduits and/or Their Absence: C(*l*,*l*+1) ∪ C'(*l*,*l*+1)

In considering vertical interactions I(*l*,*l*+1) I have shown the interacton "diffusing" through the boundaries and the "internal milieu" of the system. This may do as long as the systems are small, as in organelles and sometimes even cells and tissues. But as systems get larger, this no longer seems to do the job. We find that the vertical interactons are transmitted and guided by conduits which I have labeled C(*l*,*l*+1). The absence of such a conduit, or the complement, is C'(*l*,*l*+1). I have drawn in Figure 15 the generating set and element plans. It will be noted that the C(*l*,*l*+1) line is heavier and to the right of I(*l*,*l*+1), while C'(*l*,*l*+1) dotted and is still further to the right.

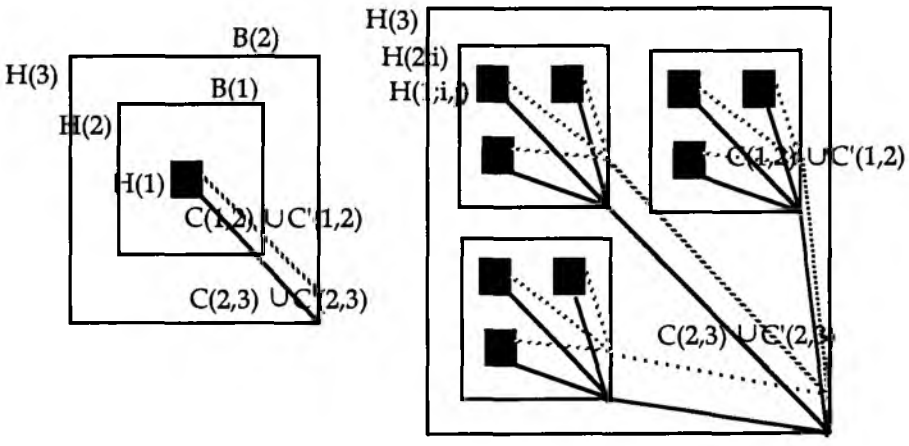


Figure 15.

In the organismal system, there are many examples of the element plan shown above. One only need look at such organ systems as the pulmonary tree (both bronchial and vascular), the arterial and venous circulations, the urinary system, and the peripheral nervous system. Similar examples may be found in the societal system and cellular system. Conduits may or may not be semi-permeable, just as the "external" boundary is. It should be noted that conduits for horizontal interactions are uncommon. I presume this is because living systems need to be very flexible and these would make them too rigid.

(5) Entrances and Exits, Open and/or Closed: E(l) ∪ E'(l)

Boundaries, B(l), up to this point, I have considered to be semipermeable. They have allowed some (though not most) elements to pass through them, because they have what I have called entrances and exits, which may be open or closed. They may or may not be associated with a conduit C(l,l+1), and vertical interactons I(l,l+1) may or may not be transmitted through them. I have labeled these E(l) (a solid "open" line) for an opening in boundary B(l), and E'(l) (a dotted "closed" line) when it is closed. It is shown on the set and element plans in Figure 16. E(l) is a property structure and there may be one or many entrances or exits in any one boundary. E(l) ∪ E'(l) may relate to I(l,l+1) ∪ I'(l,l+1) and C(l,l+1) ∪ C'(l,l+1) in a number of different ways.

These facts can be expressed in the model with statements such as I(1,2) ∩ C(1,2) ∩ E(1) ∩ B(1) ∩ H(1), which says that a vertical interacton I(1,2) connecting H(1) with other subsets of H(2) in H(3) passes through conduit C(1,2), exit E(1) in boundary B(1). Or, there may be C'(1,2) with I(1,2) ∩ E(1). Or the interaction may be prevented by E'(1). We might have an open conduit C(1,2) but no interaction I'(1,2). And then, of course, we may have several of these in a system

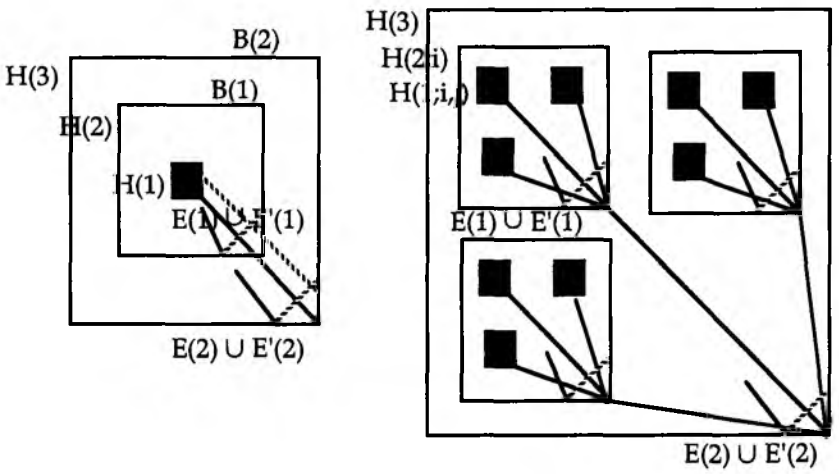


Figure 16.

at the same time. In this case, we would be summing. An organism might have open ears and/or closed eyes: E(4) ∪ E'(4).

With the following properties I do not have a property-structural symbol that I can insert into the set plan. With these, therefore, I will merely note the property values P(l) ∪ P'(l) on the set and element plans as illustrated on the plans shown in Figure 17.

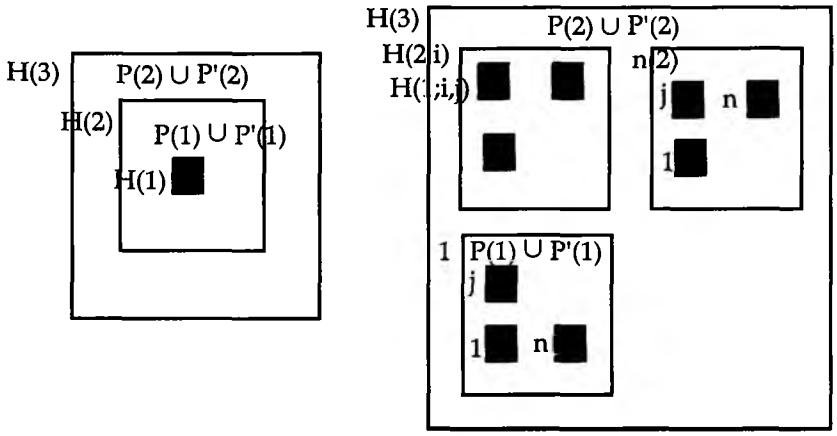


Figure 17.

(6) Growth and/or No Growth: G(*l*) ∪ G'(*l*)

This will be considered a little later in considerable detail. It suffices to say now that with this property we will be concerned with a change in the number of elements on various levels. Since I have no "structural symbol" to place on a set or element plan, I will note its presence and/or absence with a G(*l*) ∪ G'(*l*) on level *l*.

(7) Replication and/or No Replication: R(*l*) ∪ R'(*l*)

This also will be considered along with the above in more detail later. With it we are again primarily concerned with numbers of elements. Its presence and/or absence will be noted by only an R(*l*) ∪ R'(*l*) on level *l*.

(8) Support (Skeleton) and/or No Support: S(*l*) ∪ S'(*l*)

Is support present and/or not on each level of a living system? This obviously is an important property which may be difficult to quantify in any other way than stating its presence and/or absence.

(9) Self Movement (Muscle) and/or None: M(*l*) ∪ M'(*l*)

This has been a defining property of many organisms and is dispersed through many levels. What does it look like when viewed vertically? It is often related to S(*l*) ∪ S'(*l*).

(10) Mass-Energy Manufactured and/or Not: MEm(*l*) ∪ MEm'(*l*)

This has been used to define the plant kingdom and is spread through the levels of organization. Related to I(*l*,*l*+1) and C(*l*,*l*+1).

(11) Mass-Energy Absorbed and/or Not: MEa(*l*) ∪ MEa'(*l*)

An alternate way of obtaining mass and energy, being present on different levels. Also related to I(*l*,*l*+1) and C(*l*,*l*+1).

(12) Mass-Energy Stored and/or Not: MEs(l) ∪ MEs'(l)

In addition to being manufactured and absorbed, mass-energy may also be stored and/or not on a number of different levels.

(13) Facilitation and/or Inhibition: F(l) ∪ F'(l)

These in one form or another are present on most levels and would be interesting to look at vertically. Related to MEm(l), MEa(l), I(l),and I(l,l+1).

(14) Information Processing and/or No: Infp(l) ∪ Infp'(l)

Related to I(l), I(l,l+1), and C(l,l+1).

(15) Information Storage and/or No: Infs(l) ∪ Infs'(l)

(16) Charge, Positive and/or Negative: e+(l) ∪ e-(l)

While playing an important role only in the lower levels of organization, charge, like some of the other particle properties, might be at least mentioned, particularly since it plays such an important role in defining atoms (protons).

It might be helpful for some to compare the sixteen properties I have outlined to the nineteen subsystems enumerated by James Grier Miller in his book *Living Systems*. My properties and his subsystems are related by P(l)∩H(l) being a subsystem. His subsystems roughly approximate those which I have listed above. In some instances several Miller subsystems are involved in one Grant property. In a number of instances I have not listed a property corresponding to a Miller subsystem.

PROPERTIES BY GRANT	SUBSYSTEMS BY MILLER
(1) Boundaries and/or none.	2) Boundary
(2) Interactions and/or none.	12) Internal transducer 4) Distributor
(3) Vertical interactions and/or none	11) Input transducer 19) Output transducer 4) Distributor
(4) Vertical conduits and/or none.	13) Channel and net
(5) Entrances and Exits and/or none.	3) Ingestor 8)Extruder
(6) Growth and/or no growth.	6) Producer
(7) Replication and/or no replication.	1) Reproducer
(8) Support (Skeleton) and/or no support.	10) Supporter
(9) Self Movement (Muscle) and/or none.	9) Motor
(10) Mass-energy manufactured and/or not.	6) Producer
(11) Mass-energy absorbed and/or not.	6) Producer
(12) Mass-energy stored and/or not.	7) Storage
(13) Facilitation and/or inhibition.	
(14) Information processing and/or not.	14) Decoder 15) Associator 17) Decider 18) Encoder
(15) Information stored and/or not	16) Memory
(16) Charge, positive and/or negative.	

Table IV.

I have ended the induction part of this book and am ready now to launch into the abstract model itself.

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II) THE ABSTRACT HIERARCHY MODEL.

I am now ready to consider the abstract hierarchy model in some detail. I will show various representations and abstract forms of it, beginning with a three-level model and generalizing from there.

1) Some Representations of a Three-Level Model

I would like to to manipulate the three-level hierarchy model described above, showing some of its many representations. First, I will draw a three-level set floor plan and elevation plan (Figures 18 and 19 below), state their equation and show various representations of them, including how one can generate the universe of subsets. Then I will show, given a living system one wants to describe, how one determines what its specific subset description is. Finally, I will consider again the isomorphisms, the set plans and their equations.

The three level generating set **floor plan** and equation are in the left upper corner of Figure 18 (A and B). They are isomorphic or equal. H(3,**I**) represents the **universe of subsets** of these three variables, I(2) ∪ I'(2), B(1) ∪ B'(1), and I(1) ∪ I'(1). As previously described, one generates a **specific subset** algebraic description or plan H(3,**S**) by starting at the outside of the generating plan and working in, picking a property or its compliment as they come up until one reaches H(1) (route I). To generate all possible subsets and subplans, the process is repeated until all of the permutations are done, much as one expands the equation. This has been done in C and is shown with the products (leaving out constants). A binary notation is also shown. This gives the complete disjunctive normal form (complete sum of products) which contains the universe of subsets. To get the universe of subsets one must add together the different products (subplans or subsets) in all possible ways. An **alternate generating set plan** (D) which will sometimes be useful is also shown. Continuing along route I we change the eight set plans to elements plans (E) as discussed previously. This is our most concrete uninterpreted representation of the hierarchy. An alternative path to the individual elements plans is route II. This generating element plan (F) is a convenient and compact way of showing the elements and one can derive the individual element plans as one did the individual set plans. All the representations shown in Figure 18 are equal, different only in form.

If one wants to describe a particular living system H(3,S) using a subset of the above universe, one proceeds in the following manner. Any property variable $P(l) \cup P'(l)$ has four values (subsets). They are $P(l)$, $P'(l)$, $P(l) \cup P'(l)$, and $P(l) \cap P'(l) = \emptyset$, the empty subset. Looking at the data regarding the living system, one picks out the appropriate value and substitutes it into the equation (encodes). This gives us the specific subset H(3,S) (subplan) which describes our system. For instance, suppose we want to describe an organelle H(3,S) in terms of its atoms (ions) H(1) and molecules. In this case, the subset of I(2)∪I'(2) is I'(2), of B(1)∪B'(1) is B'(1), and of I(1)∪I'(1) is I(1)∪I'(1). Substituting these values into the equation gives H(3,S) = B(2)I'(2)B'(1)I(1)H(1) ∪ B(2)I'(2)B'(1)I'(1)H(1). This is marked by asterisks (*) on the appropriate subplans of Figures 18 and 19.

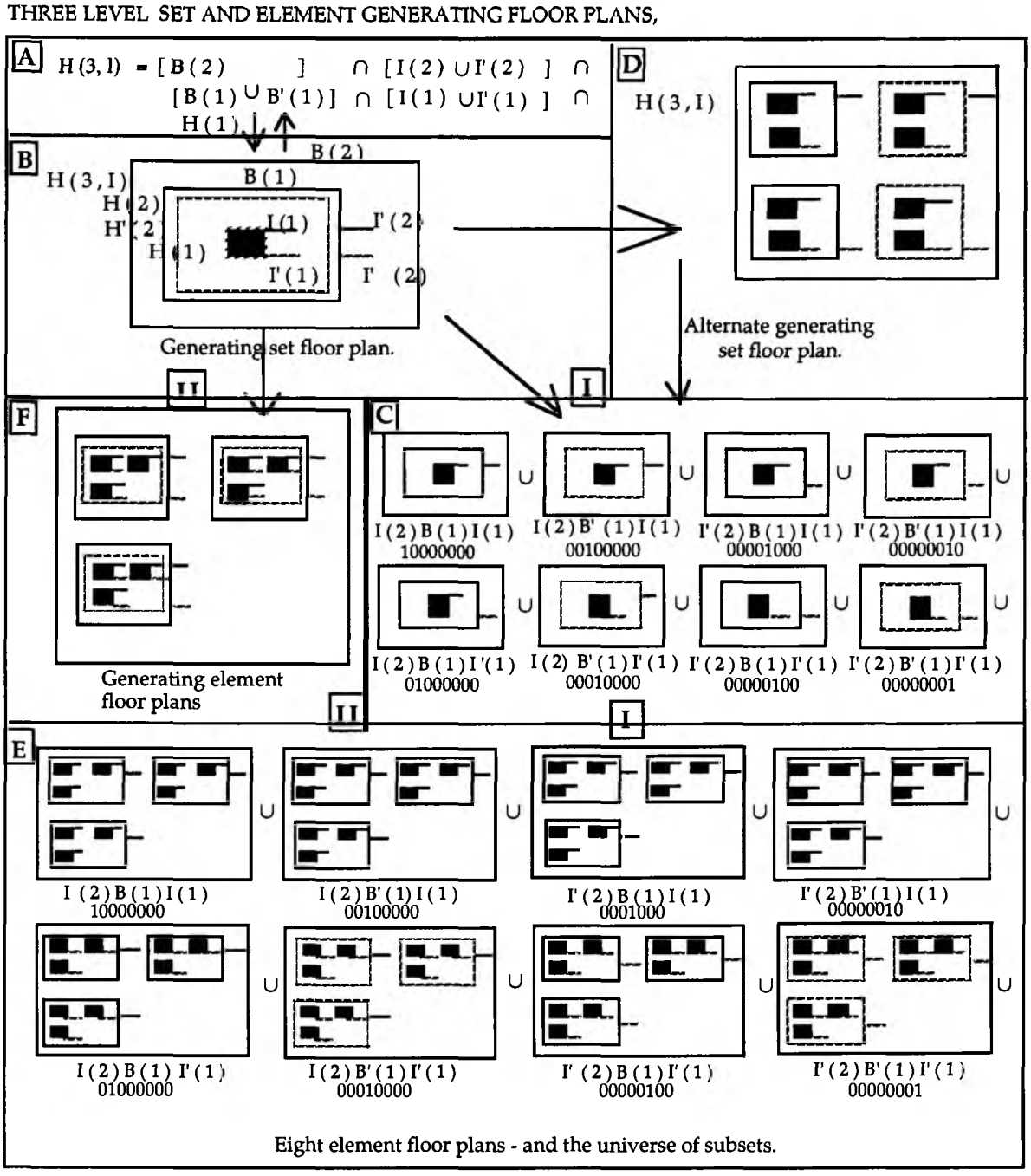


Figure 18.

We now turn to the three-level elevation plan. In Figure 19A, I have again shown the generating set floor plan H(3,I) and in 19B the isomorphic generating set elevation plan H(3,I). The figure is set up in a similar fashion to Figure 18. For route I, to generate a specific product or subset H(3,S), one starts at the top of the plan in the right upper corner and works down, picking a variable (+) or its complement (-) at each level until one reaches H(1). If we do this for all permutations, we

get the generating set plan H(3,I) as shown in C. For space reasons I have not shown the algebraic product, only the binary one (more on this later).

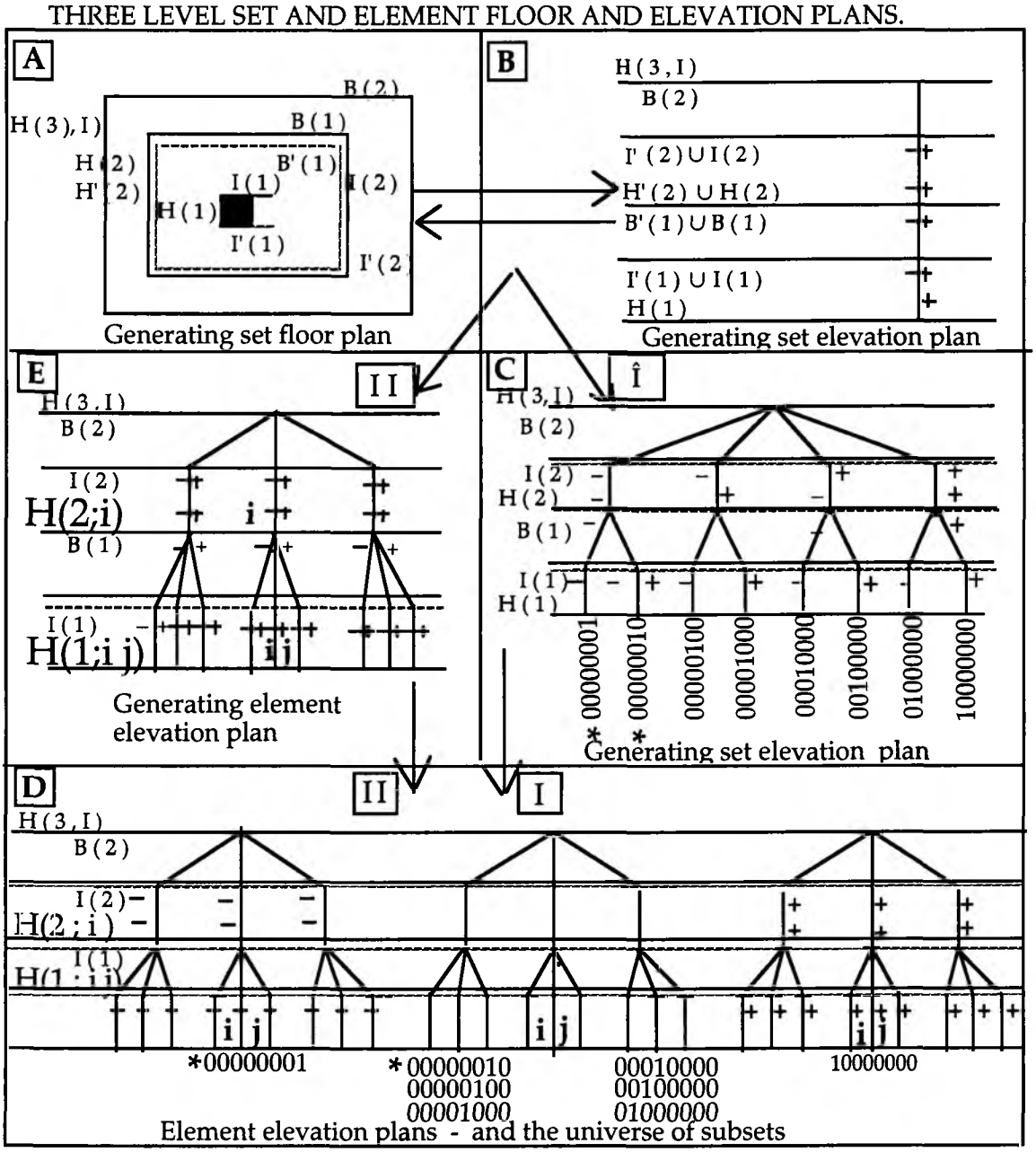


Figure 19.

If the eight products are summed, we again get the complete sum of products, which is the universe of subsets. Following along route I, we can change each product to an element plan as shown in D. For space reasons I have shown only the first and last, the others being grouped as one in the middle. Once again, these are the most concrete uninterpreted representations. As with the floor plans, one can arrive at the individual product element elevation plans by a different path--

route II. The generating set elevation plan is changed to the generating element elevation plan E, and we arrive at the bottom of the page D by the same method as before. Once again, all representations are equal and are equal to the set and element floor plans.

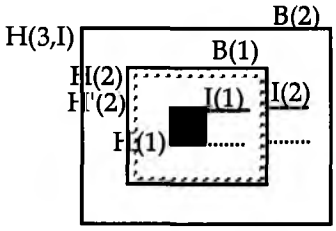
Now let us consider the equations of this three-level hierarchy. The generating equation and plans I have summarized in Figure 20a. These are all isomorphic and can be derived, one from another.

A

$$H(3,I) = [B(2)\ ] \cap [I(2) \cup I'(2)] \cap [H(2) \cup H'(2)]$$
$$H(2) \cup H'(2) = [B(1) \cup B'(1)] \cap [I(1) \cup I'(1)] \cap H(1)$$
$$H(1) = H(1)$$

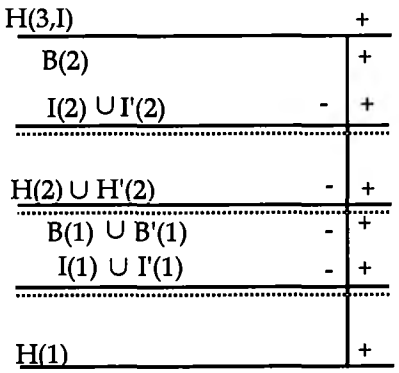
Three-level generating set equations

B



Three-level generating set floor plan

C



Three-level generating set elevation plan

Figure 20a.

If we multiply A out we get the following equation:

$$H(3;I) = B(2)\mathbf{I(2)B(1)I(1)}H(1) \cup B(2)\mathbf{I'(2)B(1)I(1)}H(1) \cup$$
$$B(2)\mathbf{I(2)B(1)I'(1)}H(1) \cup B(2)\mathbf{I'(2)B(1)I'(1)}H(1) \cup$$
$$B(2)\mathbf{I(2)B'(1)I(1)}H(1) \cup B(2)\mathbf{I'(2)B'(1)I(1)}H(1) \cup$$
$$B(2)\mathbf{I(2)B'(1)I'(1)}H(1) \cup B(2)\mathbf{I'(2)B'(1)I'(1)}H(1)$$

Equation 1.

This is the same as what appeared in the preceding set plans except that the constants have been shown also. This subset, the complete sum of products, contains all of the 256 subsets in its universe. All of them can be represented as particular floor and elevation plans similar to those shown in Figures 18 and 19.

Consider the equation generated by the set plan of Figure 20a. If we pick up H(2) ∪ H'(2) together we have the set equation as in Figure 20a.

$$
\begin{aligned}
H(3,I) &= [B(2)\ ] \cap [I(2) \cup I'(2)] \cap [H(2) \cup H'(2)] \\
H(2) \cup H'(2) &= [B(1) \cup B'(1)] \cap [I(1) \cup I'(1)] \cap H(1) \\
H(1) &= [H(1)\ ]
\end{aligned}
$$

Equations 2.

On the other hand, if we do not pick up $H(2) \cup H'(2)$, we have:

$$
\begin{aligned}
H(3,I) = [B(2)\ ] \cap [I(2) \cup I'(2)] \cap \\
[B(1) \cup B'(1)] \cap [I(1) \cup I'(1)] \cap \\
[H(1)\ ]
\end{aligned}
$$

Equation 3.

These equations are equivalent. It is in the former form that equations will usually be written, where H(3,I) may be described using either $H(2) \cup H'(2)$ or H(1) as primitive.

2) Generalization to Any Number of Levels and Properties

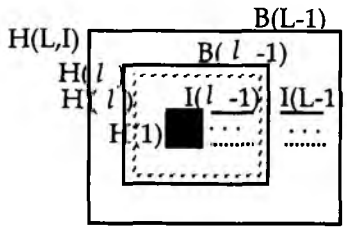
We need not confine ourselves to three-level hierarchies; we can have 4, 5,...,L levels. The generating set equation and floor and elevation plans in fact are quite similar to those above, except that, instead of l = 1, 2, and 3, we have l = 1, 2,...,L as shown in Figure 20b.

A

$$
\begin{aligned}
H(L,I) &= [B(L\text{-}1\ ] \cap [I(L\text{-}1) \cup I'(L\text{-}1)] \cap [H(L\text{-}1) \cup H'(L\text{-}1)] \\
H(l) \cup H'(l) &= [B(l\text{-}1) \cup B'(l\text{-}1)] \cap [I(l\text{-}1) \cup I'(l\text{-}1)] \cap [H(l\text{-}1) \cup H'(l\text{-}1)] \\
H(2) \cup H'(2) &= [B(1) \cup B'(1)] \cap [I(1) \cup I'(1)] \cap [H(1)\ ] \\
H(1) &= H(1)
\end{aligned}
$$

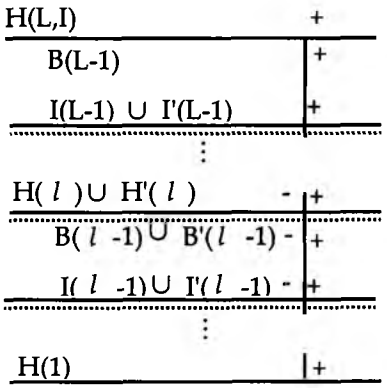
L level generating set equations

B



L level generating set floor plan

C



L level generating set elevation plan

Figure 20b.

This may be written more compactly as shown in Equation 4. In all of these equations, the independent variables (the parts) appear on the right side of the equation and the dependent variable (the whole) appears on the left.

$$H(L,I) = \bigcap_{l=1}^{L-1} \{[B(l) \cup B'(l)] \cap [I(l) \cup I'(l)] \cap [H(l) \cup H'(l)]\}$$

where $B'(L-1) = H'(1) = \emptyset$

Equation 4.

If we let L = 5 and partially multiply out, we have the equation for a five-level hierarchy which is seen so frequently in living systems. I have called this the **five-level hierarchy space equation.**

Any system.	**The hierarchy space equations.**
A Living system with	$H(5,I) = [B(4) \quad] \cap [I(4) \cup I'(4)] \cap [H(4) \cup H'(4)]$
sub-systems and it's	$H(4) \cup H'(4) = [B(3) \cup B'(3)] \cap [I(3) \cup I'(3)] \cap [H(3) \cup H'(3)]$
sub-sub-systems & it's	$H(3) \cup H'(3) = [B(2) \cup B'(2)] \cap [I(2) \cup I'(2)] \cap [H(2) \cup H'(2)]$
sub-sub-sub-systems &	$H(2) \cup H'(2) = [B(1) \cup B'(1)] \cap [I(1) \cup I'(1)] \cap [H(1)]$
sub-sub-sub-sub-systems	$H(1) = [H(1) \quad]$

Equations 5.

I would now like to generalize to any number of properties. If we let $P_i(l)$ be any property i of m properties, on any level l, the top level being L, we can generalize Equations 5 to Equation 6 below.

$$H(L,I) = \bigcap_{l=1}^{L-1} \bigcap_{i=1}^{m} [P_i(l) \cup P'_i(l)] \cap [H(l) \cup H'(l)]$$

where $H'(l) = \emptyset$

Equation 6.

To check this, one lets L = 5, m = 2, $P_1(l) = B(l)$, and $P_2(l) = I(l)$ and expands the equation. This gives us Equations 5 again.

Equation 6 is the most abstract representation of our hierarchical model, its fundamental formula. The parameter L, the number of levels being considered, will often be 5, but can assume any value we wish. The parameter m is the number of properties that we are investigating. If we let m=1, where $P_1(l) = B(l)$, we get what I have sometimes called the boundary equation. If m = 2 with $P_2(l)=I(l)$, as in the above example, we get the hierarchy space equation (Equations 5). For m=3 with $P_1(l)$ and $P_2(l)$ being $B(l)$ and $I(l)$ as above, and $P_3(l) = P(l)$, we get what I have called **the hierarchy property equation**. Since this is frequently used to investigate five-level living systems, I have expanded this to Equations 7.

$$\begin{aligned}
H(5,I-3) &= [B(4) \quad] \cap [I(4) \cup I'(4)] \cap [P(4) \cup P'(4)] \cap [H(4) \cup H'(4)] \\
H(4) \cup H'(4) &= [B(3) \cup B'(3)] \cap [I(3) \cup I'(3)] \cap [P(3) \cup P'(3)] \cap [H(3) \cup H'(3)] \\
H(3) \cup H'(3) &= [B(2) \cup B'(2)] \cap [I(2) \cup I'(2)] \cap [P(2) \cup P'(2)] \cap [H(2) \cup H'(2)] \\
H(2) \cup H'(2) &= [B(1) \cup B'(1)] \cap [I(1) \cup I'(1)] \cap [P(1) \cup P'(1)] \cap H(1) \\
H(1) &= [H(1) \quad]
\end{aligned}$$

Equations 7.

Since the universe of subsets I is contained in H(5) and m = 3, I have used the notation H(5,I-3) above, or more generally H(L,I-m). This proves to be useful at times. For instance,

$$H(5,S-3) = \mathbf{H(5,S-2)} \bigcap_{l=1}^{4} P(l) \cup P'(l)$$

Equation 8.

Thus, to arrive at the particular hierarchy property distribution for any H(5,S-3), one can use the already valued hierarchical space **H(5,S-2)** for this system and multiply by $P(l) \cup P'(l)$, valued over all four levels. This can then be done repeatedly for different properties to see how they are distributed in this particular living system.

3) The progression of hierarchy equations.

It may be helpful at this point to briefly summarize the progression of equations that we have gone through.

The generalized hierarchy equation or fundamental formula.

$$H(L,I-m) = \bigcap_{l=1}^{L-1} \bigcap_{i=1}^{m} \{[P_i(l) \cup P'_i(l)] \cap [H(l) \cup H'(l)]\}$$

where $H'(1) = \varnothing$

Equation 9.

From this equation we can obtain all of the following equations.

If we let m = 1, 2, and 3 progressively we get

a) The hierarchy boundary equation

Let m = 1, then if we let $P_1(l) = B(l)$, we have

$$H(L, I\text{-}1) = \bigcap_{l=1}^{L-1} \{[\mathbf{B}(l) \cup \mathbf{B}'(l)] \cap [H(l) \cup H'(l)]\}$$

where $B'(L-1) = H'(1) = 0$

Equation 10.

b) The hierarchy space equation (discontinuity equation).

Let m = 2; then if we let $P_2(l) = I(l)$, we have

$$H(L,I-2) = \bigcap_{l=1}^{L-1} \{[\mathbf{B}(l) \cup \mathbf{B}'(l)] \cap [\mathbf{I}(l) \cup \mathbf{I}'(l)] \cap [H(l) \cup H'(l)]\}$$

where $B'(L-1) = H'(1) = 0$

Equation 11.

c) The hierarchy property equation.

Let m = 3, then $P_3(l) = P(l)$, and we have

$$H(L,I\text{-}3) = \bigcap_{l=1}^{L-1} [B(l) \cup B'(l)] \cap [I(l) \cup I'(l)] \cap [P(l) \cup P'(l)] \cap [H(l) \cup H'(l)]$$

where $B'(L-1) = H'(1) = \varnothing$

Equation 12a.

These various forms of the basic equation will be appearing throughout the book along with the set and element floor plans and elevation plans. It will be noted again that they all are generating equations and plans, are isomorphic to one another, and are written so that they contain the universe of subsets, that is, H(L,I) ⊃ the universe of subsets (at hand). After describing (encoding) any system, we will have only one of these subsets, H(L,S), S being that particular subset. I will cover this again in a bit more detail later.

Again, Equation 5 I call the hierarchy space equation because it is used to define the levels of organization of any living system. That is, biological systems occupy both horizontal and vertical space, and this equation helps define that space(More on this later).

The set floor plan is our principal model. It is compared to nature and it generates the set elevation plan and set equations. It functions much as the Bohr atom solar system model functioned in generating the early equations of the hydrogen atom.

4) Exploring the Parameters of the Hierarchy Equation

It is helpful to construct as compact a representation of the hierarchy equation as possible. This most abstract and general equation allows one to grasp in a glance the entire hierarchy (with all of its variables and parameters) for any living system. It leads to an equation H(L,I-m) which can generate the universe of possible descriptions for any property on all levels, using $P_i(l)$. Looking at the data regarding the particular living system, one must then only fill in the values of the parameters L, n(*l*), m, and subsets $P_i(l)$ to solve for H(L,S-m), the subset description of the living system we are interested in. The equation with its **parameters** and variables is:

$$H(L) = \bigcap_{l=1}^{L-1} \bigcap_{i=1}^{m} ([P_i(l) \cup P'(l)] \cap [H(l) \cup H'(l)])$$

Equation 12b.

Interpretation of m:

m=1 : $P_1(l) = B(l)$ is used in **The hierarchy boundary equation.**

m=2 : $P_2(l) = I(l)$ is used in **The hierarchy space equation.**

m=3 : $P_3(l)$ is used in **The hierarchy property equation.**

More properties may be added to all levels of the hierarchy equation, one property at a time.

1) m is the number of properties being considered (including B(*l*) and I(*l*)). Considering only m = 1, B(*l*), we arrive at the boundary equation which defines most of the levels. Adding m = 2 adds I(l) and gives us the hierarchy space equation, which defines all levels. Properties may be added to

all levels of this system thereafter to give what I have called the hierarchy property equation. These are added and valued one at a time to keep down the complexity.

2) **L** is the top level and therefore indicates the number of levels.

3) $B'(L-1) = H'(1) = \emptyset$ (empty)

4) Subsets containing $I(l+1) \cap B'(l) \cap I'(l)$ are disallowed.

5) Subsets containing $P_i(l) \cap H(l)$ are permitted only for level l, others are disallowed and empty except for i = 1 and 2. See later for explanation of 4) and 5) above.

6) The subsets of $P_i(l)$ are $P_i(l) \cup P'_i(l)$, $P_i(l)$, $P'_i(l)$, and $\emptyset$.

7) Other properties $P_i(l)$ considered or mentioned are $P_i(l) = I(l,l+1)$, $C(l,l+1)$, $E(l)$, $G(l)$, $R(l)$, $S(l)$, $M(l)$, $MEm(l)$, $MEa(l)$, $MEs(l)$, $F(l)$, $Infp(l)$, $Infs(l)$, and $e+(l)$.

8) To change from subsets to elements, we only have to substitute in the set equation the parameter $\mathbf{n(l)}$ shown in Equation 12c.

$$H(l) = \{H(l;i) \mid i = 1,2,...,\mathbf{n(l)}\}$$

Equation 12c.

It is useful to explore the values of the parameters. The parameter L obviously establishes the number of levels or the depth of the vertical dimension of the hierarchy. Thus far, one has seen elevation plans in which L = 1, 2, 3, and 5. The parameter $n(l)$ (l= 1, 2, ...,$n(l)$), on the other hand, establishes the breadth of the horizontal dimension and is seen in element plans where $n(l)$ is enumerated, n= 1, 2, ...,$n(l)$. The parameter m is the number of properties involved in the description and thus far has been confined to m = 1, 2, 3, and occasionally 4. As one seeks to describe more completely a particular system, one adds properties (increases m) to the righthand side of the equation. The problem of reductionism is dealt with in this book by increasing m. (See Section E6.)

Thus far, I have presented twelve different representations of hierarchical algebra, the generating and individual set and element floor plans, the generating and individual set and element elevation plans, and the generating equations and individual terms of the algebra itself. In set algebra, however, there are other representations and I will turn to these in the next section.

5) Other Boolean representations

a) The Venn diagram

I would like to compare this model with other representations used in Boolean algebra. The most familiar representation is the Venn diagram shown in Figure 21. This Venn diagram is equivalent to the generating set floor plan shown on the right above. The products are all given in the middle in the algebraic as well as a binary notation that will be addressed shortly. The products are summed in all.

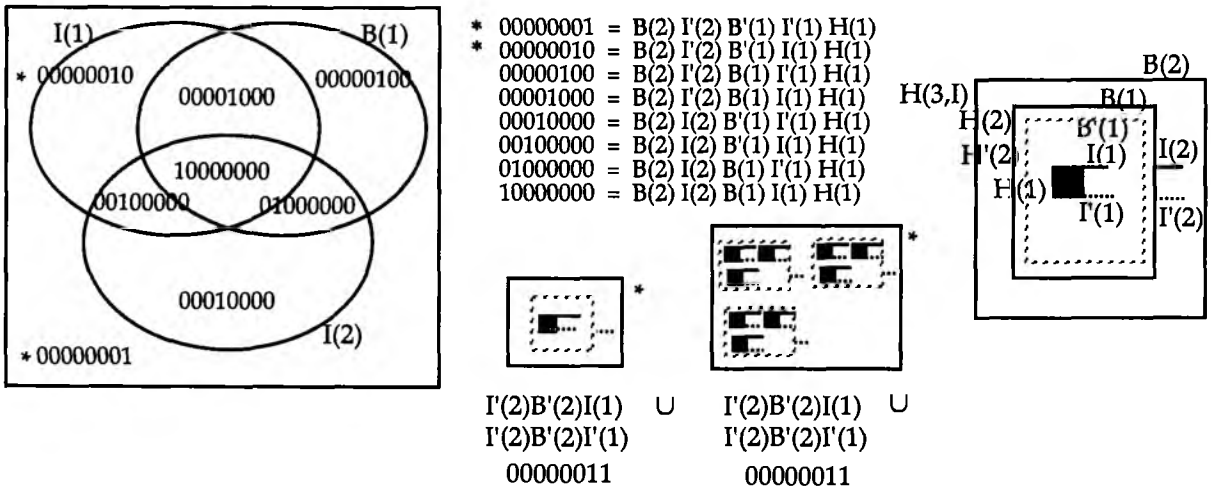


Figure 21.

possible ways to arrive at the universe of subsets H(3,I). The subset H(3,S) 00000011 or B(2)I'(2)B'(1)[I(1)∪I'(1)]H(1), as represented in the set and element plans of Figures 18 and 19, is reproduced above, marked by asterisks *.

Boolean algebra has two other representations that I will consider, as they prove to be useful in applications of hierarchy algebra. They are the truth table in binary form and the lattice. The binary notation developed in the truth table, although abstract, leads to great economy in labeling and in summing. Boolean algebra has been called a complemented distributive lattice, and, indeed, the lattice displays not only all of the subsets but also their relationships. These two representations I will illustrate using a two-level, two-variable hierarchy, because it is easier to present than the three-level, three- variable hierarchy that we have been considering. There are only $16 = 2^4$ rather than $256 = 2^8$ subset plans in its universe.

b) The truth table with binary notation

Consider the two-variable, two-level generating equation H(2,I) = [B'(1)∪B(1)] ∩ [I'(1)∪I(1)] ∩ H(1). In Figure 22, I have drawn its generating set plan on the left, and on the right are the four subplans or products which can be summed in various ways to create the universe of 16 subsets or subplans . Again I have labeled the subset plans using the binary symbolism which will be developed in Table V.

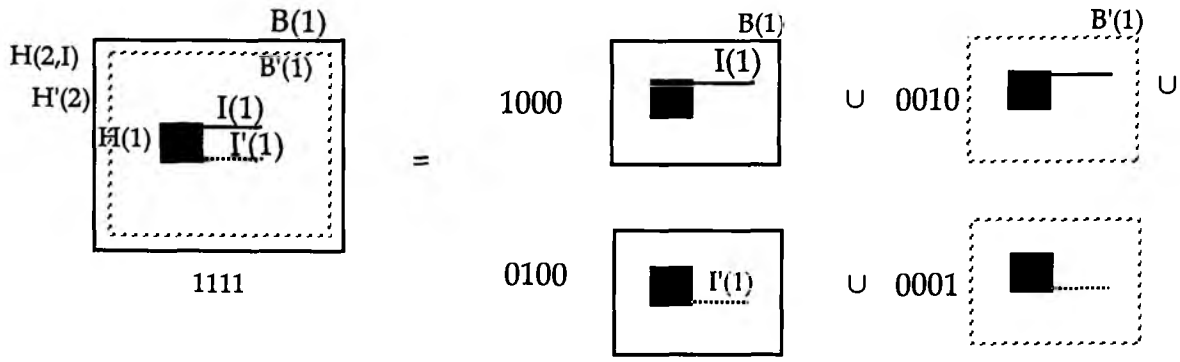


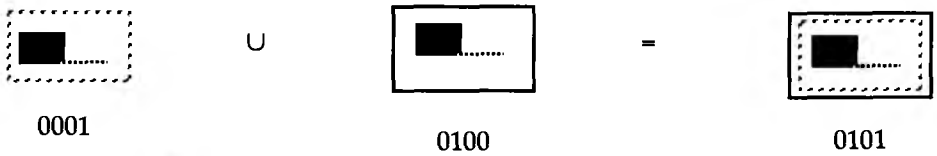
Figure 22.

In Table V, I have shown the truth table, the universe of subsets, and the genesis of the binary notation. The truth table is shown in the three minor right-hand columns of the two major columns of Table V. In the two left-hand columns of the latter, I have shown also some other representations of the algebraic form; the binary notation, the subplan representation, and a binary to decimal conversion. Using this binary notation, we can easily sum the subset products, which are then in disjunctive normal form, the form in which we need to write our set plans. The decimal notation is sometimes used for economic reasons. An example of binary summing is shown in Figure 23 below.

In both the truth table and the lattice, I have shown the subset plans, algebraic and binary subset notations. The set plans are not necessarily reduced to "monomial" subplans; some are still in a generating form, as may be the algebraic expressions. The binary form can be easily translated into the sum of products. This will, I hope, be clearer after considering the lattice.

	I(1)	B(1)			I(1)	B(1)	
0 0000 [I'(1) ∩ I(1)] ∪ [B'(1) ∩ B(1)]	1 0 1 0	1 1 0 0	[I'(1) ∩ I(1)] ∪ [B'(1) ∩ B(1)] 0 0 0 0	9 1001 [I'(1) ∩ B'(1)] ∪ I(1) ∩ B(1)]	1 0 1 0	1 1 0 0	[I'(1) ∩ B'(1)] ∪ [I(1) ∩ B(1)] 1 0 0 1
1 0001 I'(1) ∩ B'(1)	1 0 1 0	1 1 0 1	I'(1) ∩ B'(1) 0 0 0 1	10 1010	1 0 1 0	1 1 0 0	I(1) 1 0 1 0
2 0010 I(1) ∩ B'(1)	1 0 1 0	1 1 0 0	I(1) ∩ B'(1) 0 0 1 0	12 1100 B(1)	1 0 1 0	1 1 0 0	B(1) 1 1 0 0
4 0100 I'(1) ∩ B(1)	1 0 1 0	1 1 0 0	I'(1) ∩ B(1) 0 1 0 0	7 0111 I'(1) ∪ B'(1)	1 0 1 0	1 1 0 0	I'(1) ∪ B'(1) 0 1 1 1
8 1000 I(1) ∩ B(1)	1 0 1 0	1 1 0 1	I(1) ∩ B(1) 1 0 0 0	11 1011 I(1) ∪ B'(1)	1 0 1 0	1 1 0 0	I(1) ∪ B'(1) 1 0 1 1
3 0011 B'(1)	1 0 1 0	1 1 0 1	B'(1) 0 0 1 1	13 1101 I'(1) ∪ B(1)	1 0 1 0	1 1 0 0	I'(1) ∪ B(1) 1 1 0 1
5 0101 I'(1)	1 0 1 0	1 1 0 1	I'(1) 0 1 0 1	14 1110 I(1) ∪ B(1)	1 0 1 0	1 1 0 0	I(1) ∪ B(1) 1 1 1 0
6 0110 [I(1) ∩ B'(1)] ∪ [I(1) ∩ B(1)]	1 0 1 0	1 1 0 1	[I(1) ∩ B'(1)] ∪ [I(1) ∩ B(1)] 0 1 1 0	15 1111 [I'(1) ∪ I(1)] ∩ B'(1) ∪ B(1)]	1 0 1 0	1 1 0 0	[I'(1) ∪ I(1)] ∩ [B'(1) ∪ B(1)] 1 1 1 1

Table V.



[B'(1) I'(1)] ∪ [B (1) I'(1)] = [B' (1) ∪ B(1)] I' (1)

Figure 23.

c) The Lattice.

The binary notation is a more compact way of expressing the subsets, even though they are abstract and need translation. Both of these come to play when we show the final representation of hierarchy algebra, the lattice as shown in Figure 24. In this lattice, I have shown each subset in algebraic, binary, and set plan form. The algebraic and set plans may or may not be simplified. Those that are are still in a generating form and may be "multiplied out" to their constituent plans and products if so desired. To the left, under **Layer,** I have given the number of products summed needed to describe a given subset (plan). Thus, on the third layer we need three summed products to describe each plan or system. These may then of course be simplified and have been so in the algebraic and set plan representations on the lattice. It will be noted that the "top" subset contains all of the others and in fact is our generating equation for the universe of subsets and subplans. The usual meet (intersection) and join (union) relationships hold in this complemented, distributive lattice. Of particular interest is the fact that the first layer of monomials is what I have previously referred to as products or terms. In this context, summing products to get the universe of subsets means moving up the lattice, adding one product per layer. Moving down the lattice involves intersecting the terms.

Having discussed the binary notation and lattices, I can now present in a clearer fashion the notation which differentiates the equations generating the universe of subsets, and that which describes a specific system subset. As noted previously, I have used H(2,I) to represent the universe of subsets of this two-level hierarchical system. H(2,S) represents that particular subset which describes the two-level living system that we are interested in. Obviously H(2,I) > H(2,S).

Consulting the lattice of Figure 24, one notes that the subset 15 at the top, $[I'(1)\cup I(1)]\cap[B'(1)\cup B(1)]$, when multiplied out, gives the complete sum of products. That is, it contains the four products of Figure 12. When these are summed in all possible ways, we generate the universe of all 16 subsets, as seen in Table V and Figure 24. Furthermore, each of these subsets has a binary and decimal label. Thus we have a convenient way of specifying exactly which subset we are referring to. I illustrate this below, referring to Figure 24.

H(2,S) = H(2,15), the specific subset, but
H(2,I) = $H(2)\cup H'(2)\supset H(2,15)\cup H(2,14)\cup H(2,13)\cup\ldots, \cup H(2,2)\cup H(1,1)\cup H(1,0)$, i.e., H(2,I) contains the universe of subsets.
H(2,S) = H(2,12), the specific subset, but
$H(2,12)\supset H(2,12)\cup H(2,8)\cup H(2,4)\cup H(1,0)$, i.e., H(2,12) contains these subsets.

The decimal notation, though abstract, is very economic.

I might also note at this time that not all of the 16 subsets meet the interactional and boundary criteria for elevating the system to a higher level. Subsets 0000 (0) and 0001 (1) have neither interactions between elements nor boundaries around them. Thus H(1,0) and H(1,1) cannot qualify to become H(2,0) or H(2,1). The rest do qualify.

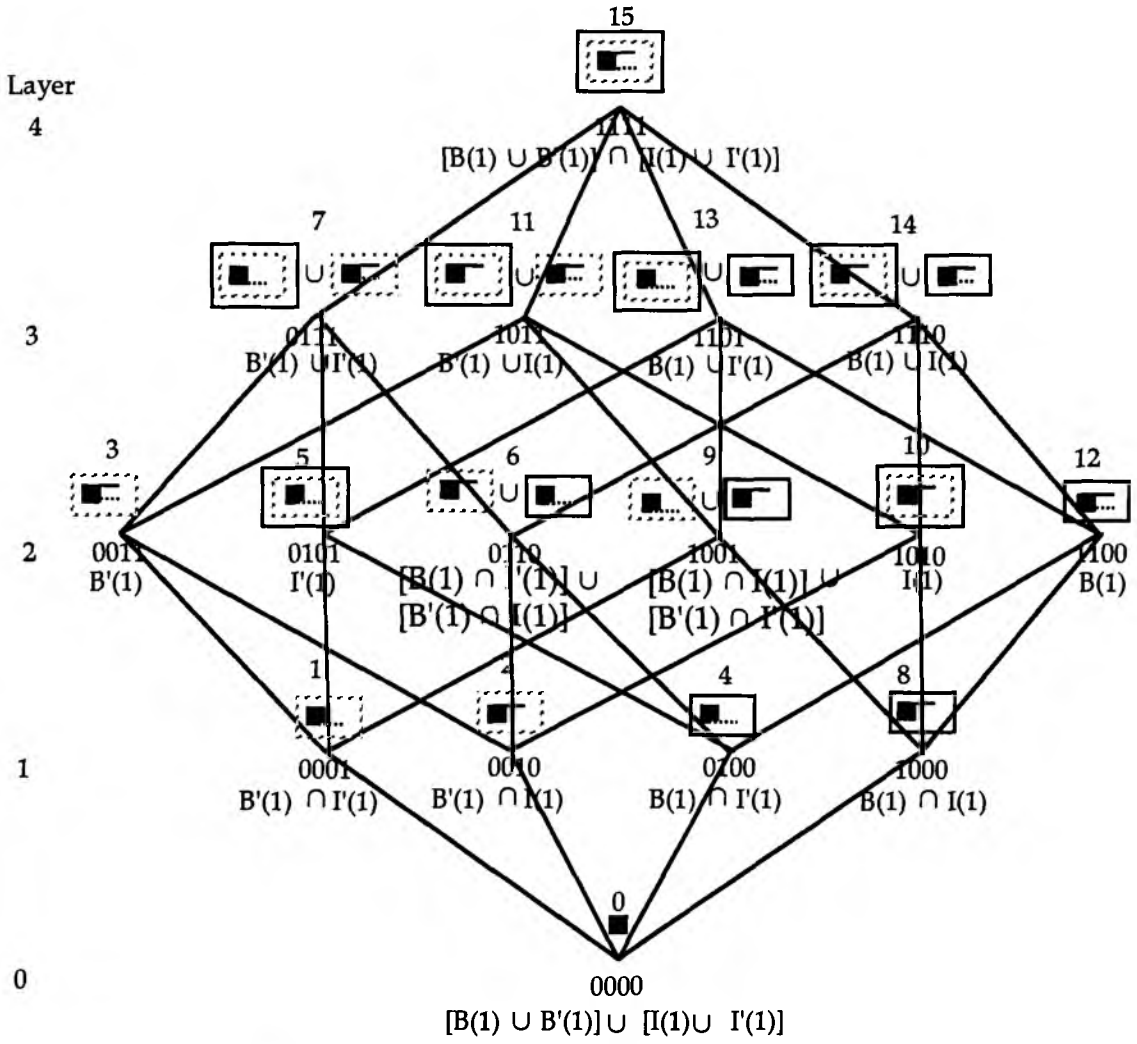


Figure 24.

Equations 12a and 12b comprise the fundamental formula of this model, and it would be worthwhile to put it into English. The formula relates the elements and properties on one level of organization P∩H(*l*) with an element H(*l*+1) on the next level above it. This can then be repeated for all L levels of the hierarchy.

We have explored various aspects of the abstract model, the architectural plans, Venn diagrams, lattices, and hierarchy equations. Now I would like to turn to some uses of the model. First I want to show that these can be encoded to describe any living system. More particularly, one can order the data regarding the living system showing its hierarchical and vertical structure in a way that seems intuitively true. After this I will use the model to classify some living systems (organisms) and show how the model can be used to predict data. Finally, I will consider some implications of the model.

III) DEDUCTIONS USING THE HIERARCHY MODEL

Using the model to describe, order, classify, and predict data of the real world

I would now like to consider how one uses the model to work back to the real world. That is, by encoding reality data into the model, we hope to be able to deduce various other reality data about the real world. But first, a word about description, ordering, classification, and prediction.

It is absolutely imperative that in encoding data into our model, we arrive at a description, which, when decoded, accurately describes the system in which we are interested. As long as we do not manipulate the model between encoding and decoding, we should have no difficulty with this. That is, we are getting out only what we put in; and if the data are encoded precisely and correctly, they should give an accurate (although of course an incomplete) decoded description of the living system. Since the model is hierarchically arranged, the data in the description will be hierarchically ordered. This, in itself, is of great interest since it gives us a new way to look at the data.

Using the hierarchically ordered descriptions to classify living systems (e.g., organisms) is a step up. Once again, if we are not manipulating the model, this should present no particular difficulties. It does position us, however, to get to the final use of the model. By manipulating the model according to the rules of set algebra, we should be able, if the model is a good one, to predict new data regarding living systems. This gives us the opportunity to check on the value, validity, and utility of the model.

A) DESCRIPTIONS USING THE HIERARCHY MODEL

Since the descriptive function of the model is so central, I will describe a variety of living systems. For simplicity the first example will use only the $B(l) \cup B'(l)$ set variable. For the next several examples I will use the space model, which uses the boundary and interactional properties, places the elements on different levels, and relates them one to another. Later I will add other properties to the space model to produce the property model. This shows how the properties distribute themselves over the various levels. Since encoding the data into the models is so important, I will describe this again and then illustrate it in each description. Usually, I will decode again almost immediately. Let me begin by reviewing the model.

1) Review of Hierarchy Model and Fundamental Formula

It will be remembered that the fundamental formula for any hierarchical model with L levels and n properties is as shown in Equation 11.

$$H(L,I\text{-}n) = \bigcap_{l=1}^{L-1} \bigcap_{i=1}^{n} \{[P_i(l) \cup P'_i(l)] \cap [H(l) \cup H'(l)]\}$$

$$\text{where } H'(1) = \emptyset$$

Equation 11.

The boundary equation had only one set variable, $B(l) \cup B'(l)$. Its equation is the same as below with $I(l) \cup I'(l) = 1$ and ignored. The space equation has two properties, $B(l)$ and $I(l)$. Its equation is below as Equation 12.

$$H(L,I\text{-}2) = \bigcap_{l=1}^{L-1} \{[B(l) \cup B'(l)] \cap [I(l) \cup I'(l)] \cap [H(l) \cup H'(l)]\}$$

where $B'(L\text{-}1) = H'(1) = \emptyset$

Equation 12.

For a five-level living system, these become Equations 13.

Any system.	The hierarchy space equation.
A Living system	$H(5,I\text{-}2) = [B(4) \quad] \cap [I(4) \cup I'(4)] \cap [H(4) \cup H'(4)]$
sub-systems	$H(4) \cup H'(4) = [B(3) \cup B'(3)] \cap [I(3) \cup I'(3)] \cap [H(3) \cup H'(3)]$
sub-sub-systems	$H(3) \cup H'(3) = [B(2) \cup B'(2)] \cap [I(2) \cup I'(2)] \cap [H(2) \cup H'(2)]$
sub-sub-sub-systems	$H(2) \cup H'(2) = [B(1) \cup B'(1)] \cap [I(1) \cup I'(1)] \cap H(1)$
sub-sub-sub-sub-systems	$H(1) = [H(1) \quad]$

Equations 13.

From Equations 13, one can draw the rather crowded five-level generating set floor plan shown in Figure 25. Both the equation and the plan imply the universe of subsets.

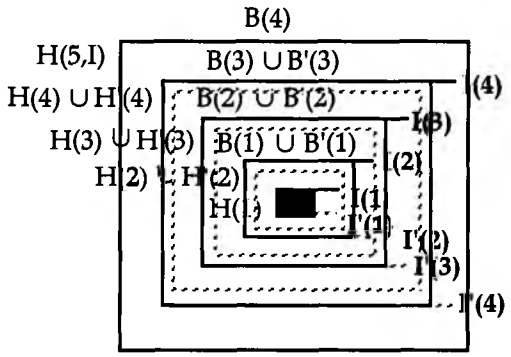


Figure 25.

2) Encoding and Decoding

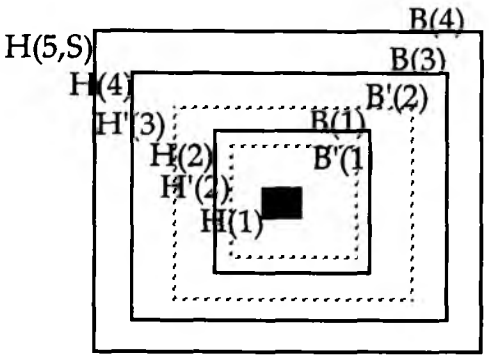
It will be recalled that every property variable has four subsets, $P(l)$, $P'(l)$, $P(l) \cup P'(l)$, and $P(l) \cap P'(l)$. To encode the property into the model, one looks at the data regarding the system one wishes to describe, picks the appropriate subset, and enters this in hierarchy notation into the model. This is then done for the properties $B(l)$ and $I(l)$ for all levels. Once the model is encoded to decode or interpret, one simply substitutes back the data in the place of the hierarchy notation. From the universe of subsets H(5,I-2) one has, in the encoded model, gone to a particular subset H(5,S-2) which describes the system. One has solved for S.

3) Using the Space Model to Describe Some Simple Systems

I will use this model to describe some relatively simple systems as an example of how the model is used. The first example is the description of a societal system. In this, I will use only the boundary variable (the boundary equation). In the second example, I will use both $B(l)$ and $I(l)$ and describe any and all molecules, and then a particular molecule. In my final example, I will again use the space equation to locate all the elements in a eukariotic cell.

a) A particular social system

In the first example of how the model is used, I would like to describe a particular (and artificial) social system. In this example I am using only the boundary model, no additional properties. Keeping in mind the general boundary equation and plan H(5,I) of Figure 25 (without the interactional property I(*l*)), in Figure 26, I have identified (in fantasy) the various B(*l*)∪B'(*l*) (independent) variables observed in this society and placed their values in the set plan. Now solve for the dependent variable H(5,S).



The generating set plan for a specific five-level nation, the variables being defined and valued.

H(5,S)= This nation
H(4)= Community
H'(3)= Extended family
H(2)= Nuclear family
H'(2)= Homeless family
H(1)= Person

B(4)= National bound.
B(3)= Community limits
B'(2)= No compound boundary
B(1)= House
B'(1)= No house

Figure 26.

5) H(5,S) = [B(4)] ∩ H(4)
4) H(4) = [B(3)] ∩ H'(3)
3) H'(3) = [B'(2)] ∩ H(2) ∪ H'(2)
2) H(2) ∪ H'(2)= [B(1) ∪ B'(1)] ∩ H(1)
1) H(1) = [H(1)]

Equations 14.

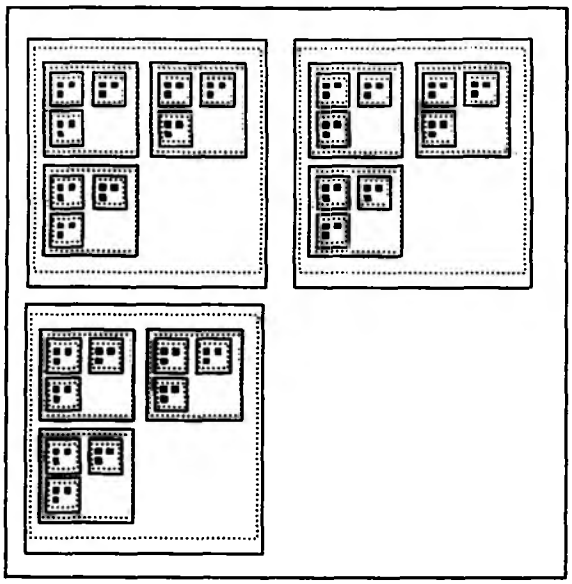
This nation and its communities have well-defined boundaries; and since all families and persons that live in the nation also live in communities, there is no B'(3). In these communities there are no extended family compounds (boundaries), therefore we have B'(2), and thus H'(3) is the extended family. However, there are nuclear families that have houses, B(1), as well as those that are homeless, B'(1), thus we have H(2) ∪ H'(2). These values have been encoded into Figure 25 and Equations 13 to get Figure 26 and Equations 14.

Equations 14 give the subset description of this rather simplified society. Other subsets would describe different societies. One could write these equations as the single equation

$$H(5,S) = B(4) \cap B(3) \cap B'(2) \cap [B(1) \cup B'(1)] \cap H(1).$$

However, I find that writing more complex hierarchy equations in a vertical rather than horizontal manner makes them easier to think about. Therefore, I will write the hierarchy equations vertically, as in Equations 14 above. This also allows us to insert other properties on the right side of the equation for each level.

In this example, to get more specific and concrete, we would want to identify the elements of each subset of this society. In Figure 27, I have shown the generating element plans for both the universe of subsets and the particular social system that we are considering. The left-hand generating plan H(5,I) has as its universe of subsets $2^3 = 8$ terms and 2^8 or 256 subsets. The right-hand plan, this social system H(5,S), is solved and is described by the one subset noted. Our most concrete description involves enumerating and decoding (identifying) the elements at each level of this particular social system.

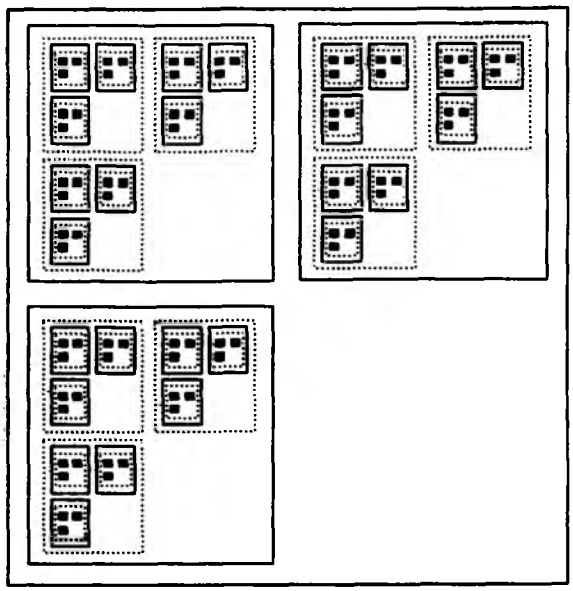


The universe of subsets.

$$H(l+1;I) = \bigcap_{l=1}^{4} [B(l) \cup B'(l)] \cap H(1) \qquad \text{where } B'(4) = 0$$

2^3 = 8 terms

2^8 = 256 subsets



Subset describing this social system.

$$H(5;S) = B(4)B(3)B'(2)B(1)H(1) \cup B(4)B(3)B'(2)B'(1)H(1)$$

2 terms

1 subset

Figure 27.

If, in the description of the society noted above, we were to find a person in the nation but not in a family or community, we would need to add another term to our description, getting a new subset. This new term would be B(4)B'(3)B'(2)B'(1)H(1).

b) Molecules

Basic chemistry is the bedrock on which living systems lie, and hierarchically it is simpler than living systems. Thus it is to molecules that I apply my second interpretation. If we let nucleons be our primitive elements, then molecules form a three-level hierarchy, that is, the first level is nucleons, the second is atoms, and the third is molecules. The generating set floor and elevation plans and equations are shown in Figure 28 and Equations 15.

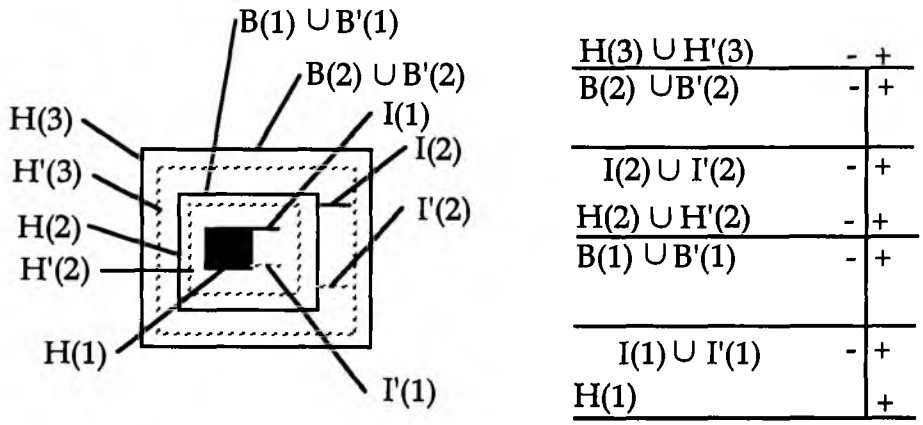


Figure 28.

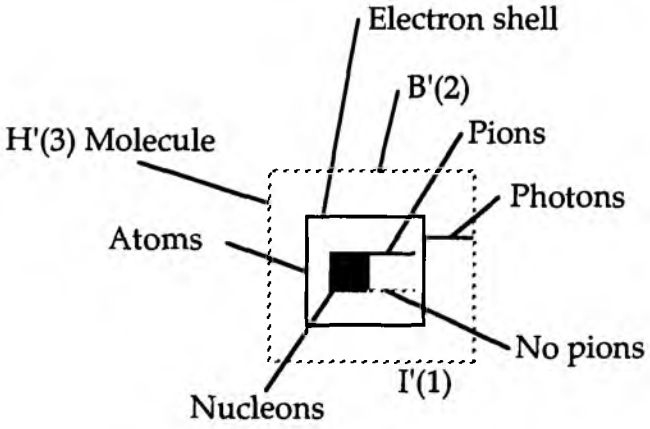
$$H(3) \cup H'(3) = [B(2) \cup B'(2)] \cap [I(2) \cup I'(2)] \cap H(2) \cup H'(2)$$
$$H(2) \cup H'(2) = [B(1) \cup B'(1)] \cap [I(1) \cup I'(1)] \cap H(1)$$
$$H(1) = [H(1)\ ]$$

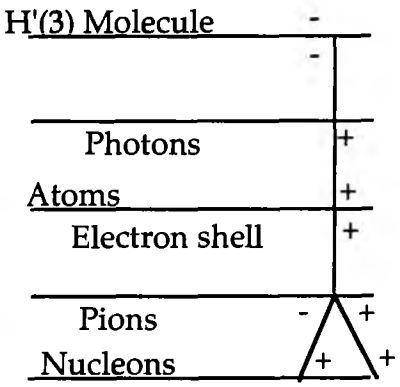
Equation 15.

If we then look at the data on molecules, we find that of the nucleons H(1), some may interact with one another through the strong force, I(1) (pions), and some may not, I'(1). They are surrounded by an electron shell, B(1), that acts as a boundary (to negatively charged particles) and form atoms, H(2). The atoms interact with one another through photons, I(2), and thus form a molecule, which we will call H'(3,S). If we substitute these values into the model, we get Figure 29 and Equations 16. These are the set plans and equations, the hierarchy space, for all molecules.

The set floor plan of all molecules.

The set elevation plan of all molecules.





The set floor plan of all molecules.

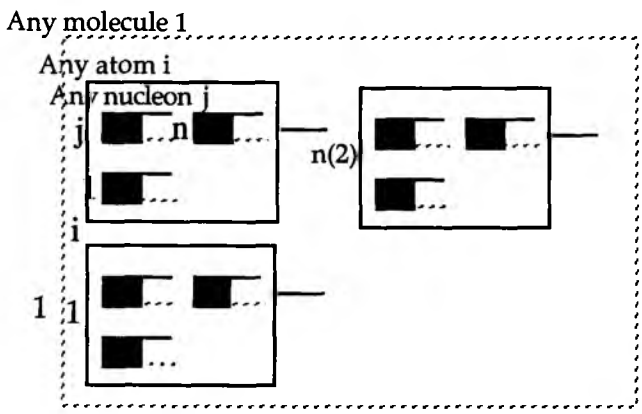
The set elevation plan of all molecules.

Figure 29.

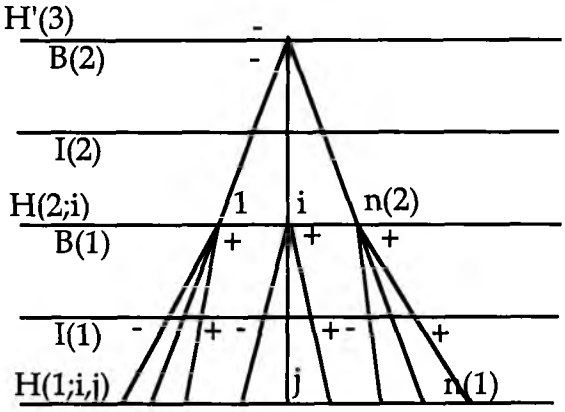
$$H'(3,S) = [\quad B'(2)] \cap [I(2) \quad] \cap H(2)$$
$$H(2) = [B(1) \quad] \cap [I(1) \cup I'(1)] \cap H(1)$$
$$H(1) = [H(1) \quad]$$

Equations 16.

The set equation and plans are qualitative and describe the hierarchical structure of all small molecules. To differentiate between molecules we must move to the element plans and equations which are quantitative. I have shown these below.



The element floor plan



The element elevation plan

H'(3,S) = Any small molecule
H(2;i) = the i^{th} atom, i = 1,2,...,n(2)
H(1;i,j) = the j^{th} nucleon in the i^{th} atom, j = 1,2,...,n(1;i)

Figure 30.

$$H'(3,S) = [\quad B'(2;1)] \cap [I(2) \quad] \cap H(2;i)$$
$$H(2;i) = [B(1;i) \quad] \cap [I(1) \cup I'(1)] \cap H(1;i,j)$$
$$H(1;i,j) = [H(1;i,j) \quad]$$

Equations 17.

I have not completely labeled the variables again, as the element plan and tree becomes too crowded and the interpretations are clear from the above. In using the words "small molecule," I mean to differentiate this from macro-molecules, which, as will be seen later, are at the next higher level. The set equations and plans describe all molecules. In considering elements, however, we are able to differentiate different molecules from each other. That is, if H'(3,S) is any particular molecule, then H(2;i), where i = 1,2,...,n(2), is any atom within this molecule, and each atom, in turn, is defined by its nucleons H(1;i,j) where j = 1,2,...,n(1;i). Clearly, we can describe many different molecules depending upon what n(2) and n(1;i) are.

c) The carbon dioxide molecule

For instance, consider the carbon dioxide molecule O-C-O, where we have atoms 1,2,and 3, or n(2) = 3. These interact through chemical bonds. The two oxygen atoms, H(2;1) and H(2;3), each have 16 nucleons, n(1;1) = n(1;3) = 16, while the carbon atom, H(2;2), is made up of 12 nucleons, n(1;2) = 12.

What these element plans do not do, however, is to allow us to differentiate between protons and neutrons, and to identify atoms, we must be able to label the protons. Thus we must

introduce another property to the two that we already have. This property, of course, is charge. If we let e+(l) be positive charges on elements on the lth level, then e'(l) indicates negative charges on elements on the lth level, and e+(l) ∪ e'(l) means that there are both positive and negative elements on this level. If we let e+(l) ∪ e'(l) = 1, then elements do not have the property or its complement, being neutral elements. I could have inserted this property also into the space equation and in this way identified the protons and atoms.

d) Macromolecules

At the other end of this hierarchy, and not shown above, we have H'(4,S) or macro-molecules. It is with macromolecules that we finally arrive at the hallmark of living systems. These I will consider later. It suffices to say that the elements of macromolecules (e.g., proteins) are certain small molecules (e.g., amino acids), and the interactions, I(3), or bonds between macromolecules, are often of a different nature from I(2) bonds. Once again, there is no clear-cut B(3); so we have B'(3) and therefore H'(4,S) macro-molecules.

There are many other properties of living systems that can be arranged hierarchically on this framework, and in later models I will be considering some of them. However, adding variables to the model increases its complexity exponentially, and one rapidly loses the ability to "visualize" the whole. That is, if m is the number of variables, then 2 to the mth is the number of terms (products), and, 2^{2^m}, is the number of possible subsets in the universe of subsets. And, of course, each subset may have from none to billions of elements. Because of this, I will add only one or, at the most, two categories of properties to the hierarchy space equation at any one time. However, by looking at different views, we may get a glimpse of the whole.

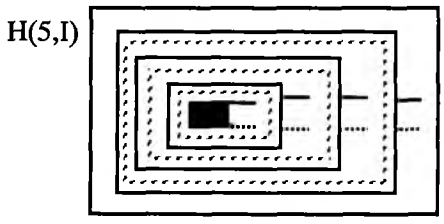
Before turning to the property models, I would like to consider one more living system in considerable detail. This is one of our simplest living systems, the eukaryotic cell, and even though we have inserted only two categories of variables (boundaries and interactions), the picture we arrive at is quite illustrative.

4) A More Complex Example: a Description of any Eukariotic Cell

I would like to partially describe and order hierarchically any and all eukaryotic cells using the five-level hierarchical set plan and equation.

a) The general five-level set space plan and equation

We begin with the general set plan and space equation of H(5,I) shown in Figure 31. In this, there are seven variables and 2^7 or 128 products. These, in turn, contain 2^{128} or >10 to the 38th subsets in this universe. Only one, however, describes the cell.

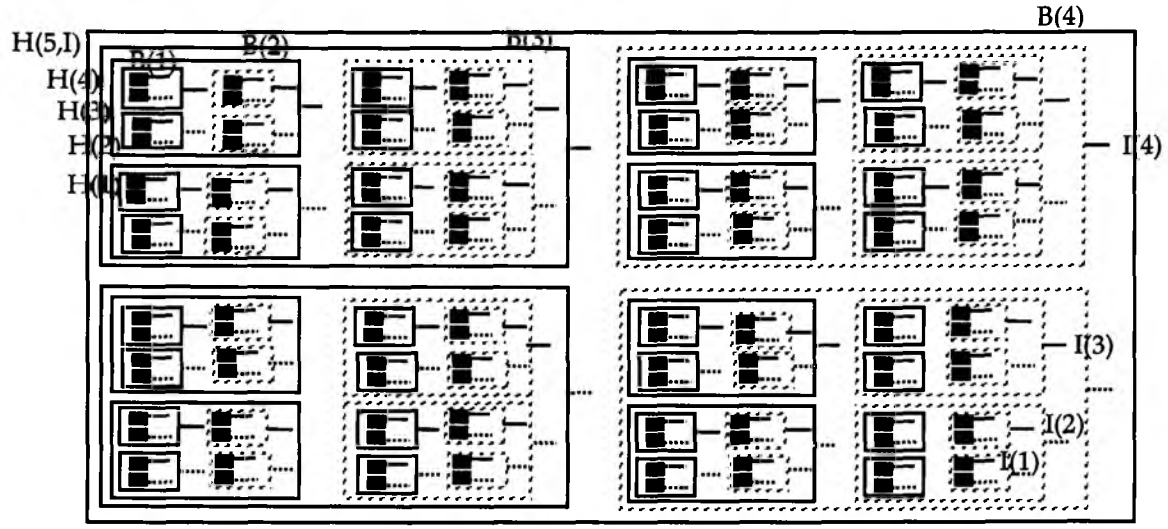


H(5,I) =[B(4)] ∩ [I(4)∪I'(4)] ∩ [H(4)∪H'(4)]
H(4)∪H'(4)=[B(3)∪B'(3)] ∩ [I(3)∪I'(3)] ∩ [H(3)∪H'(3)]
H(3)∪H'(3)=[B(2)∪B'(2)] ∩ [I(2)∪I'(2)] ∩ [H(2)∪H'(2)]
H(2)∪H'(2)=[B(1)∪B'(1)] ∩ [I(1)∪I'(1)] ∩ [H(1)∪H'(1)]
H(1) =[H(1)]

Figure 31.

b) The alternate generating set plan

In the two previous examples, I have moved directly from the set plan and equations to the element plans and equations following Route II of Figure 18. This time, however, I will first generate all of the subsets, following Route I (the alternate generating set plan). This allows me to make a more complete description of the cell.



7 variables, 128 terms, and > 10^{38} subsets

Figure 32.

Consider the general set equation and plan of Figure 31. We have 128 products in the complete sum of products. These may be arrived at by expanding the equation or by moving inward on the set plan in all possible ways and taking the union of these. These can be put in an alternative set plan, as shown in Figure 32. One "reads" this plan by starting at the outside and moving in to H(1), picking up the subsets B(*l*) and I(*l*) or their complements at each level, whatever is present. These, in turn, are summed to give the complete sum of products. This contains the universe of subsets.

c) The first cut of subsets

Now we want to solve for H(5,S), the one subset that describes the cell. How do we do this? First we look at the data regarding the real cell for the various variables and find the following values.

B(4)--a cell membrane is present, but there is no cell without a membrane, so no B'(4).

B(3) ∪ B'(3)--Organelle membranes present. But some smaller elements are not enclosed with an organelle membrane, so B'(3) also.

B'(2)--Since there are no macromolecular boundaries present.

B'(1)--Since there are no molecular boundaries present.

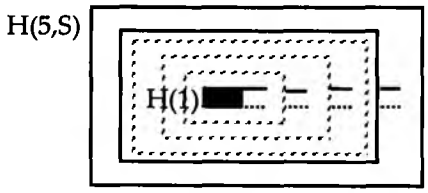
I(4) ∪ I'(4)--Interactions between organelles, present and/or not.

I(3) ∪ I'(3)--Interactions between macromolecules, present and/or not.

I(2) ∪ I'(2)--Secondary chemical bonds, both present and/or not.

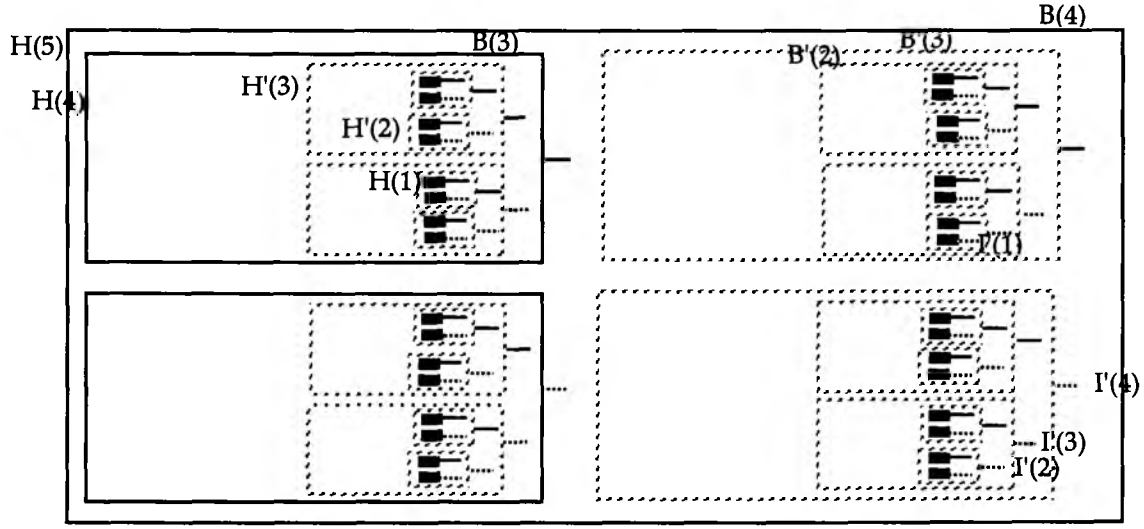
I(1) ∪ I'(1)--Primary chemical bonds, both present and/or not.

Using these, we can draw a preliminary set plan and equation. See Figure 33.



H(5,S) = [B(4)] ∩ [I(4)∪I'(4)] ∩ [H(4)∪H'(4)]
H(4)∪H'(4) = [B(3)∪B'(3)] ∩ [I(3)∪I'(3)] ∩ [H'(3)]
H'(3) = [B'(2)] ∩ [I(2)∪I'(2)] ∩ [H'(2)]
H'(2) = [B'(1)] ∩ [I(1)∪I'(1)] ∩ [H(1)]
H(1) = [H(1)]

Figure 33.



Five variables, thirty-two terms, and greater than four billion subsets.

Figure 34.

If we repeat the process that we did for the previous alternate set plan, we find that it now looks like Figure 34. We have decreased dramatically the number of subsets, having only somewhat more than 4 billion to deal with. However, with the following line of reasoning we can decrease this to the one subset description we are looking for.

d) The second cut of subsets

Our fundamental space equation was as seen previously:

$$H(L,I) = \bigcap_{l=1}^{L} \{[\mathbf{B}(l) \cup \mathbf{B'}(l)] \cap [\mathbf{I}(l) \cup \mathbf{I'}(l)] \cap H(l)\}$$

where B'(L-1) = H'(1) = Ø

Equation 18

Consider the middle (**highlighted**) portion of this showing the two variables. This is multiplied out as shown in Figure 35. The level is raised to a higher level in three instances, but not in the fourth. There are two subsets of this fourth instance. In the one preceded by I'(l+1) the subset is allowed, but it remains on the same level. But in the one preceded by I(l+1) ,this subset is disallowed by definition of I(l+1), which is defined for H(l+1) but not for H(l). That is, because B'(l)∩I'(l), I(l+1) is applied to H(l), for which it is not defined.

Figure 35.

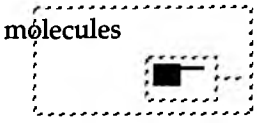
	@ B(l)	B'(l)
I(l)	B(l) ∩ I(l)	B'(l) ∩ I(l) *
I'(l)	B(l) ∩ I'(l)	B'(l) ∩ I'(l) #

@ B(l) ∩ I(l) ∩ H(l) ∪ B(l) ∩ I'(l) ∩ H(l) = H(l+1) and the level is raised.
* B'(l) ∩ I(l) ∩ H(l) = H'(l+1) and the level is raised.
\# **B'(l) ∩ I'(l)** ∩ H(l) and the level is not raised.
Of the products in the last set, there are two subsets.
1 **I'**(>l+1) ∩ **B'(l) ∩ I'(l)** ∩ H(l) is allowed, the level is not raised,
2 but **I**(>l+1) ∩ **B'(l) ∩ I'(l)** ∩ H(l) is disallowed, therefore empty by definition of I(>l+1), which is defined for H(>l+1) but not for H(l).

Figure 35.

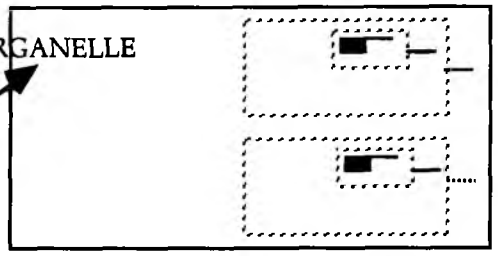
Thus we are able to exclude all those subsets which have in them a I(>l+1)∩B'(l)∩I'(l)∩H(l). This gives us our cell as shown in Figure 38, where the subsets are interpreted as in Figures 36 and 37 below.

H(1) = **B'**(1) I'(1) H(1)



H'(2) = B'(1) I(1) H(1)

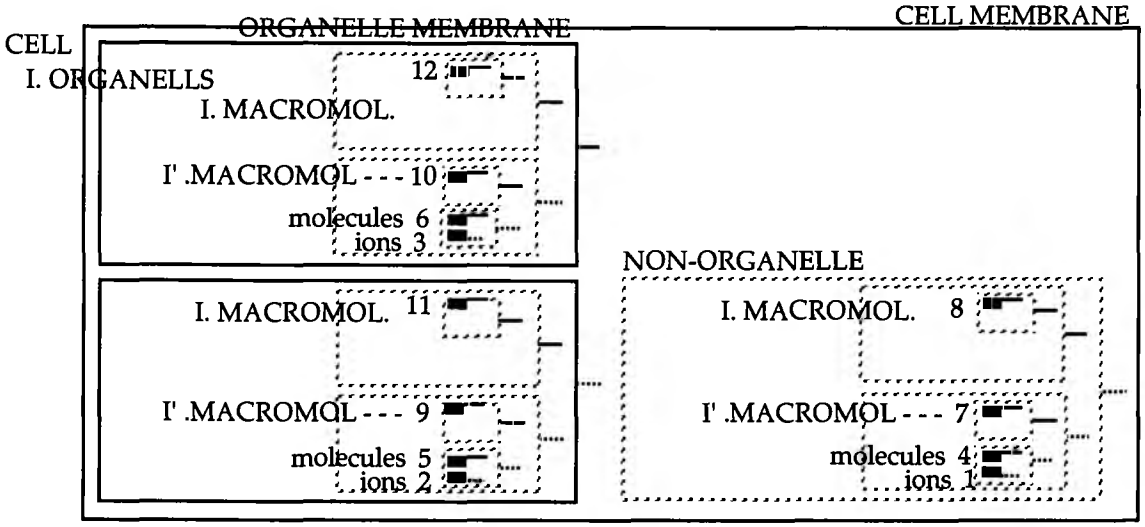
Figure 36.



H'(3) = B'(2) I(2) B'(1) I(1) H(1)
H(4) = B(3) [I(3) ∪I'(3)]B'(2) I(2) B'(1) I(1) H(1)

Figure 37.

e) H(5,S), the cell set space floor plan and equation



Five variables, twelve terms, and one subset - which describes the cell.

Figure 38.

In this figure, organelles and macromolecules can be both interacting (I) or not (I'), and are so indicated. Furthermore, the elements that are not in an organelle but in the cell are indicated as non-organelle. This set plan for the cell is almost a picture of any eukariotic cell, showing how the elements are grouped. Using this, or the calculation shown in Appendix B, we have solved for H(5,S).

	level	term number
H(5,S)=B(4)I'(4)B'(3)I'(3)B'(2)I'(2)B'(1)I'(1) ∩ **[H(1)]** ∪	1	1
B(4)I'(4) ∩ [B(3)I'(3)B'(2)I'(2)B'(1)I'(1)] ∩ **[H(1)]** ∪	4,1	2
B(4)I(4) ∩ [B(3)I'(3)B'(2)I'(2)B'(1)I'(1)] ∩ **[H(1)]** ∪	4,1	3
B(4)I'(4)B'(3)I'(3)B'(2) ∩ **[I'(2)B'(1)I(1)H(1)]** ∪	2	4
B(4)I'(4) ∩ [B(3)I'(3)B'(2)] ∩ **[I'(2)B'(1)I(1)H(1)]** ∪	4,2	5
B(4)I(4) ∩ [B(3)I'(3)B'(2)] ∩ **[I'(2)B'(1)I(1)H(1)]** ∪	4,2	6
B(4)I'(4)B'(3)I'(3) ∩ **[B'(2)I(2)B'(1)I(1)H(1)]** ∪	3	7
B(4)I'(4)B'(3)I(3) ∩ **[B'(2)I(2)B'(1)I(1)H(1)]** ∪	3	8
B(4)I'(4) ∩ [B(3)I'(3)] ∩ **[B'(2)I(2)B'(1)I(1)H(1)]** ∪	4,3	9
B(4)I(4) ∩ [B(3)I'(3)] ∩ **[B'(2)I(2)B'(1)I(1)H(1)]** ∪	4,3	10
B(4)I'(4) ∩ [B(3)I(3)] ∩ **[B'(2)I(2)B'(1)I(1)H(1)]** ∪	4,3	11
B(4)I(4) ∩ [B(3)I(3)] ∩ **[B'(2)I(2)B'(1)I(1)H(1)]**	4,3	12

Equation 19.

The terms in this description are numbered arbitrarily on the right and are shown on the set plan. The highlighted subsets are the ions, molecules, or macromolecules (on the first, second, or third levels), while the underlined portion shows whether these subsets are in organelles (fourth level) or not, being just in the cell. It should be noted once again that there were more than 10^{38} possible subsets in the universe of subsets, but only one of these gives the set description of the cell.

The 12 actual terms which describe any cell are summarized below.

1---Atoms (Ions), in the cell but not in organelles

4---Molecules in the cell but not in organelles

7,8---Macromolecules, interacting and non-interacting, not in organelles but in the cell

2,3---Atoms (Ions), in interacting and non-inter. organelles.
5,6---Molecules in interacting or non-interacting organelles.
12,11,10,9---Interacting or non-interacting macromolecules in interacting or non-interacting organelles

Thus we have described the biological space of a eukaryotic cell. All of the elements of the cell have been identified and placed in the cell with their boundary and interactional relationships to each other spelled out.

f) Specific cells and their element plans and equations

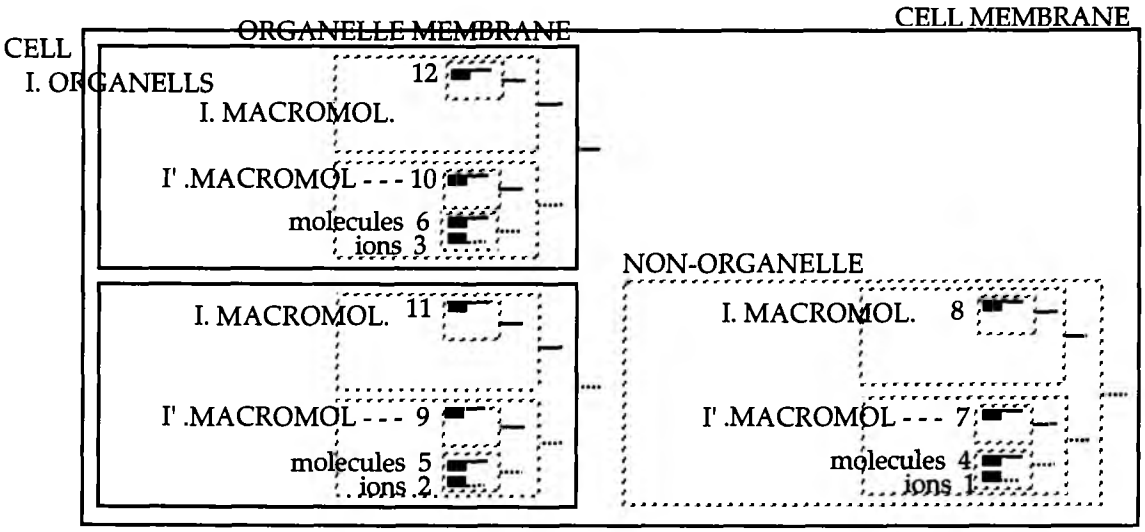
A more concrete description and one which is more cell-specific, which will vary from cell to cell, is the element description or equations. These are arrived at by substituting in the set equation in Figure 33, the elements H(*l*;i,j...) of the set H(*l*). The appropriate limits are noted. This gives us Equations 20 below.

$$H(5,S) = [B(4) \quad] \cap [I(4) \cup I'(4)] \cap [H(4) \cup H'(4)]$$
$$\{H(4;i)\} \cup \{H'(4;i')\} = [\{B(3;i)\} \cup \{B'(3;i')\}] \cap [I(3) \cup I'(3)] \cap H'(3)$$
$$\{H(3;i,j)\} = [\{B'(2;i',j')\}] \cap [I(2) \cup I'(2)] \cap H'(2)$$
$$\{H'(2;i',j',k')\} = [\{B'(1;i'j'k')\}] \cap [I(1) \cup I'(1)] \cap H(1)$$
$$\{H(1;i,j,k,l)\} = H(1)$$

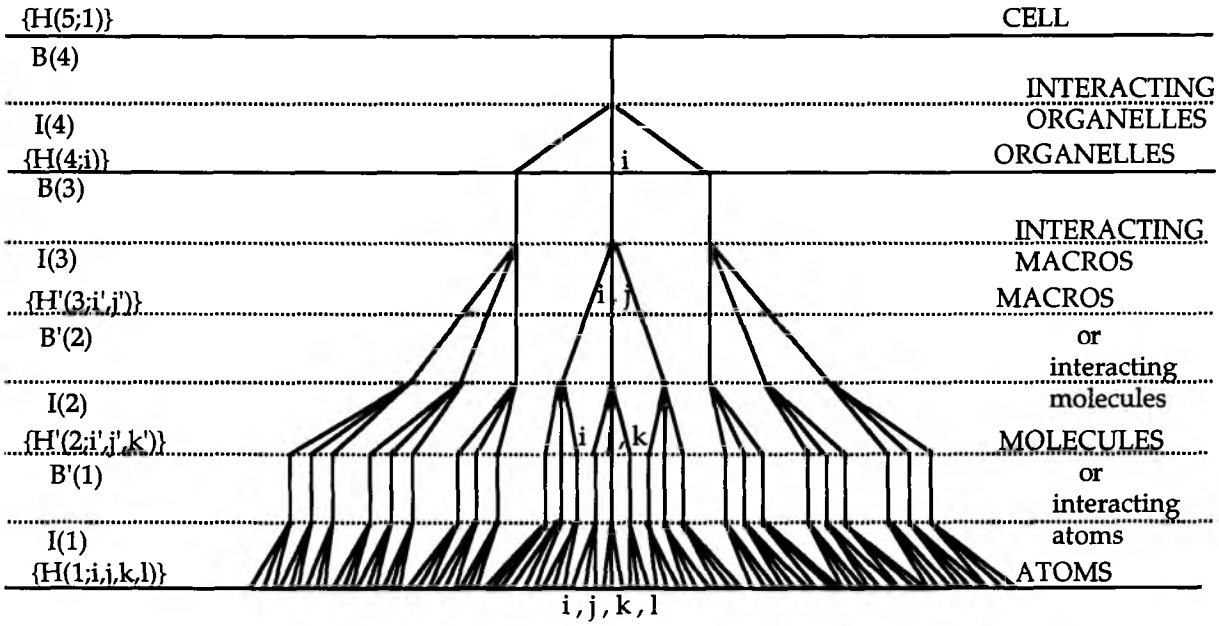
where $i = 1,2,...,n(1)$ $\quad i' = 1,2,...,n'(1)$
$j = 1,2,...,n(i)$ $\quad j' = 1,2,...,n'(i')$
$k = 1,2,...,n(i,j)$ $\quad k' = 1,2,...,n'(i',j')$
$l = 1,2,...,n(i,j,k)$ $\quad l' = 1,2,...,n'(i',j',k')$

Equations 20.

To show complete element plans of the cell, one would have to show all 12 element floor and elevation plans, which takes too much space. Therefore, just for illustrative purposes, I have shown only the set and element floor plans and equations, and the element elevation plan of term 12. To interpret this, one must enumerate all of the atoms, in all of the molecules, in all of the macromolecules, in all of the organelles in the cell. This, of course, would be a herculean, if not impossible, project.



Five variables, twelve terms, and one subset - which describes the cell.
Figure 39.



An element elevation plan

Figure 40.

If we had shown the element equations and plans for all 12 terms, we would have located in their proper places all of the individual atoms, molecules, macromolecules, and organelles in the cell.

B) ORDERING PROPERTIES HIERARCHICALLY

I would now like to show how the hierarchy space model can be used as a framework on which to hang properties, one at a time. This brings us to the hierarchy property equation. Perhaps this can be most easily understood by using a simple example. Let us go back to the set description of the eukariotic cell. It will be recalled that we started with the general five-level equation H(5,I) and substituted into it the B(l) and I(l) values found in any and all eukariotic cells. We then multiplied out, putting the equation in the form of the sum of products, removed the disallowed (empty) subsets, and ended up with a space equation for H(5,S) which consisted of one subset, the sum of 12 products, each of which located a particular subset of elements (atoms, molecules, macromolecules, organelles, etc.) within the hierarchy. The hierarchy space equation H(5,S) is shown again below.

$$
\begin{aligned}
H(5,S) &= [B(4)\ ] \cap [I(4) \cup I'(4)] \cap [H(4) \cup H'(4)] \\
H(4) \cup H'(4) &= [B(3) \cup B'(3)] \cap [I(3) \cup I'(3)] \cap [\ H'(3)] \\
H'(3) &= [\ B'(2)] \cap [I(2) \cup I'(2)] \cap [\ H'(2)] \\
H'(2) &= [\ B'(1)] \cap [I(1) \cup I'(1)] \cap [H(1)\ ] \\
H(1) &= [H(1)\ ]
\end{aligned}
$$

where $I(l+1) \cap B'(l) \cap I'(l)$ is disallowed or is empty.

Equations 21.

If we wish then to look at a particular property $P(l) \cup P'(l)$, we add it to the space equation and get the hierarchy property equation below.

$$
\begin{aligned}
H(5,S) &= [B(4)\ ] \cap [I(4) \cup I'(4)] \cap [P(4) \cup P'(4)] \cap [H(4) \cup H'(4)] \\
H(4) \cup H'(4) &= [B(3) \cup B'(3)] \cap [I(3) \cup I'(3)] \cap [P(3) \cup P'(3)] \cap [\ H'(3)] \\
H'(3) &= [\ B'(2)] \cap [I(2) \cup I'(2)] \cap [P(2) \cup P'(2)] \cap [\ H'(2)] \\
H'(2) &= [\ B'(1)] \cap [I(1) \cup I'(1)] \cap [P(1) \cup P'(1)] \cap [H(1)\ ] \\
H(1) &= [H(1)\ ]
\end{aligned}
$$

where $I(l+1) \cap B'(l) \cap I'(l)$ is disallowed and is empty.

Equations 22.

1) Conduits Within the Eukaryotic Cell

Suppose the property I wish to investigate is interlevel conduits $C(l,l+1) \cup C'(l,l+1)$ within the cell. If we substitute this in the hierarchy equation for $P(l) \cup P'(l)$ and then look at the cell, we find what I have shown in Figure 41 and Equation 23.

There are no interlevel conduits on the first and second levels. That is, there are none at the atomic-molecular or molecular-macromolecular levels, so C'(1,2) and C'(2,3) (which I have not shown in Figure 41 for space reasons). There may or may not be conduits between organelles and interacting macromolecules (I don't know); so $C(3,4) \cup C'(3,4) = 1$. On the fourth level, however, there may be intracellular microtubules C(4,5) connecting some organelles with the outside but not connecting C'(4,5) others. That is, some interlevel interactons I(4,5) may have a conduit C(4,5), while others may not C'(4,5). Thus our hierarchical model for these properties in the cell becomes what is shown below.

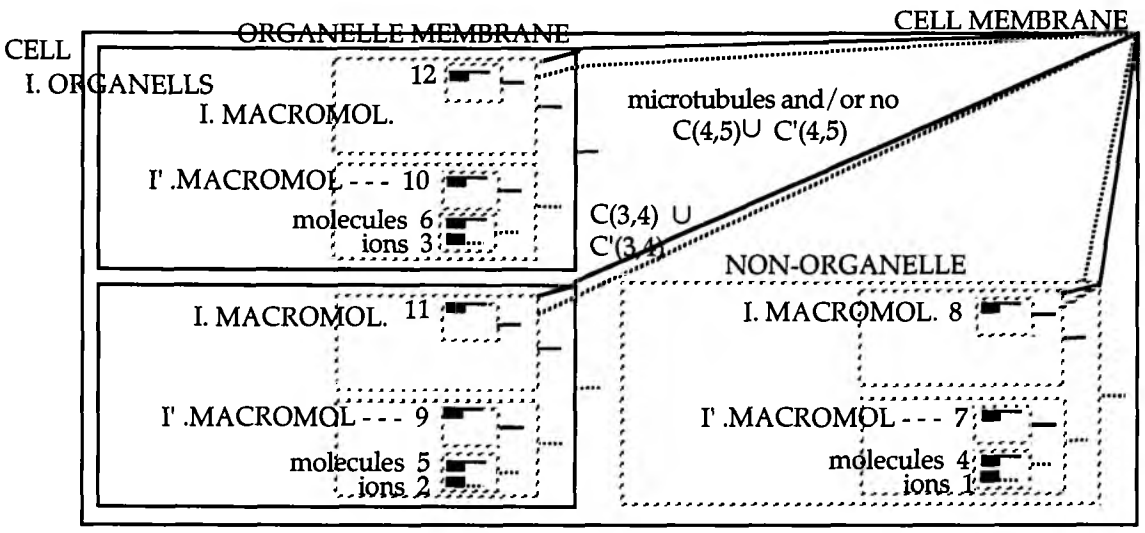


Figure 41.

$$
\begin{aligned}
H(5,S) &= [B(4)\ ] \cap [I(4) \cup I'(4)] \cap [C(4,5) \cup C'(4,5)] \cap [H(4) \cup H'(4)] \\
H(4) \cup H'(4) &= [B(3) \cup B'(3)] \cap [I(3) \cup I'(3)] \cap [\ 1\ ] \cap [\ H'(3)] \\
H'(3) &= [\ B'(2)] \cap [I(2) \cup I'(2)] \cap [\ C'(2,3)] \cap [\ H'(2)] \\
H'(2) &= [\ B'(1)] \cap [I(1) \cup I'(1)] \cap [\ C'(1,2)] \cap [H(1)\ ] \\
H(1) &= [H(1)\ ]
\end{aligned}
$$

where $I(l+1) \cap B'(l) \cap I'(l)$ is disallowed and is empty.

Equations 23.

Another way of arriving at Equations 23 is as follows. Recall Equation 8 on page 40. We have determined the particular subset H(5,S-2) that describes the space that the cell occupies (see Equation 21). To the right side of this, we then tack on the encoded $C(l,l+1) \cup C'(l,l+1)$ property. This gives us Equation 23. One can do the same then for any other property.

$$H(5,S\text{-}3) = H(5,S\text{-}2) \cap \bigcap_{l=1}^{4} C(l,l+1) \cup C'(l,l+1)$$

Equation 24.

In the following sections, I will consider other properties of living systems which can be arranged hierarchically in the space model. In order to simplify, I may not always consider all of the possible subsets, but I don't feel that this will be detremental or subtract from the argument.

2) Growth

Growth and replication are sometimes difficult to separate. I will consider them separately, however, remarking when they come together. I will consider each on a level-by-level analysis. First, the notation: Let $G(l)$ and $R(l)$ be the growth and replication, respectively, on level l. Then $G'(l)$ and $R'(l)$ indicate the absence of these on these levels. For a five-level system, the basic hierarchy property equation is Equation 25 below.

$$H(l+1) \cup H'(l+1) = \bigcap_{l=1}^{4} \{[B(l) \cup B'(l)] \cap [I(l) \cup I'(l)] \cap [P(l) \cup P'(l)] \cap [H(l) \cup H'(l)]\}$$

where $B'(4) = H'(1) = \emptyset$

Equation 25.

Let $P(l) \cup P'(l) = G(l) \cup G'(l)$. Then, in general, if the set $H(l) \cup H'(l)$ increases in size, we say that we have $G(l)[H(l) \cup H'(l)]$, or growth at level l, whereas if $H(l) \cup H'(l)$ does not increase in number, then $G'(l)[H(l) \cup H'(l)]$ or no growth at this level. If a subset of $H(l) \cup H'(l)$ increases and another subset does not, then we have $[G(l) \cup G'(l)] \cap [H(l) \cup H'(l)]$, or both growth and non-growth at this level. If we don't know the state of $G(l)$, then, since $G(l) \cup G'(l) = 1$, we can ignore it.

Now let **$P(l) \cup P'(l) = R(l) \cup R'(l)$**. Then, in the same way, if $H(l) \cup H'(l)$ replicates, or the elements split in two (or more), then $R(l)[H(l) \cup H'(l)]$ shows the replication at this level. If they don't, then $R'(l)[H(l) \cup H'(l)]$ shows no replication. Again, we may have $[R(l) \cup R'(l)] \cap [H(l) \cup H'(l)]$ if one subset replicates and another does not. If we don't know the state of $R(l)$, then, since $R(l) \cup R'(l) = 1$, we can ignore it.

There are two ways in which the elements of $H(l)$ can increase in number and grow: by being added to or by dividing. There is only one way in which $H(l)$ can replicate, and that is by dividing. In this model, growth on a given level is defined as an increase in the number of elements on this level, whereas replication is defined as an element splitting into two or more elements on this level.

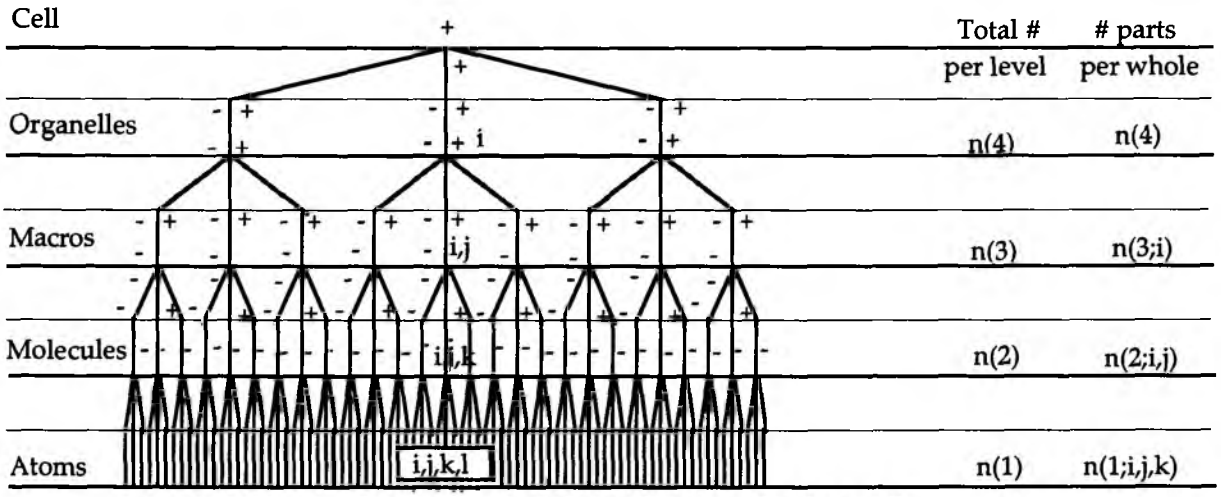
a) Growth of a cell

To illustrate this more concretely, consider the five-level system that was discussed previously, a cell (see Figure 38 and Equation 19). The space equation had 12 terms, each describing one subset of elements. These may be condensed to the following:

$H(1) = H(1,1) \cup H(1,2) \cup H(1,3)$	are all subsets of **Ions** within the cell, all in different locations.
$H'(2) = H'(2,4) \cup H'(2,5) \cup H'(2,6)$	are all subsets of **molecules** within the cell, all in different locations
$H'(3) = \bigcup_{S=7}^{12} H'(3,S)$	are all subsets of **macromolecules** in different locations
H(4) = the subset of **organelles**	within the cell, interacting and not.

The generating element elevation plan for the cell space is seen below in Figure 42. As seen on the right, on the first level there are n(1) atoms in this cell, n(2) molecules, n(3) macromolecules, and n(4) organelles. On the other hand, there are n(1;i,j,k) atoms in a molecule, n(2;i,j) molecules in a macromolecule, n(3;i) macromolecules in an organelle and n(4) organelles in the cell. As the cell grows, we have growth at all levels. Since the number of atoms, n(1), increases though ingestion, we have G(1)H(1). The number of simple molecules, n(2), increases through ingestion or synthesis from atoms below, so we have G(2)H'(2). The number of macromolecules, n(3), grows by ingestion

and by synthesis from simple molecules, therefore G(3)H'(3). Organelles number n(4) and grow by synthesis, therefore G(4)H(4). Thus we have G(l)[H(l)∪H'(l)], growth on all levels. On the other hand, there are periods during the life of a cell when there is no growth on any level, when n(l) is not increasing. Thus we also have G'(l)[H(l) ∪ H'(l)]. If we consider both, we end up with [G(l) ∪ G'(l)] ∩ [H(l) ∪ H'(l)], where l = 1,2,3,4. H(3), H(2), and H'(1) are all empty.



The generating element elevation plan for a cell.

Figure 42.

b) Growth of a macromolecule

If we focus down on any one element we find the same. Consider a macromolecule. In the element plan below (Figure 43) one can say that, as this molecule grows, it adds simple molecules. That is, n(2) gets larger, and as it does so does n(1). If H'(3) were decoded to a protein, then it has n(2) amino acids and, as the protein grows, n(2) increases. As the amino acids increase, so do the atoms n(1).

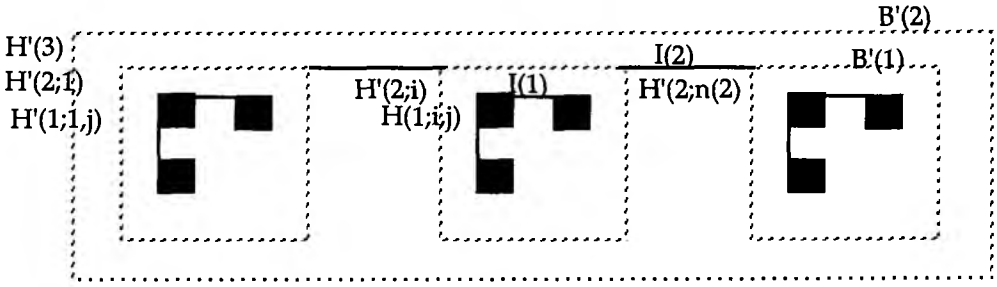


Figure 43.

Decode Figure 43 as

H'(3) = A protein
H'(2;i) = Any amino acid i, i = 1,2,...,n(2)
H(1;i,j) = Any atom j in an amino acid i, j = 1,2,...,n(1;i)
I(1) = A primary chemical bond.
I(2) = A secondary chemical bond.
n(2) = The number of amino acids molecules in the protein.

n(1;i) = The number of atoms in any amino acid molecule i.

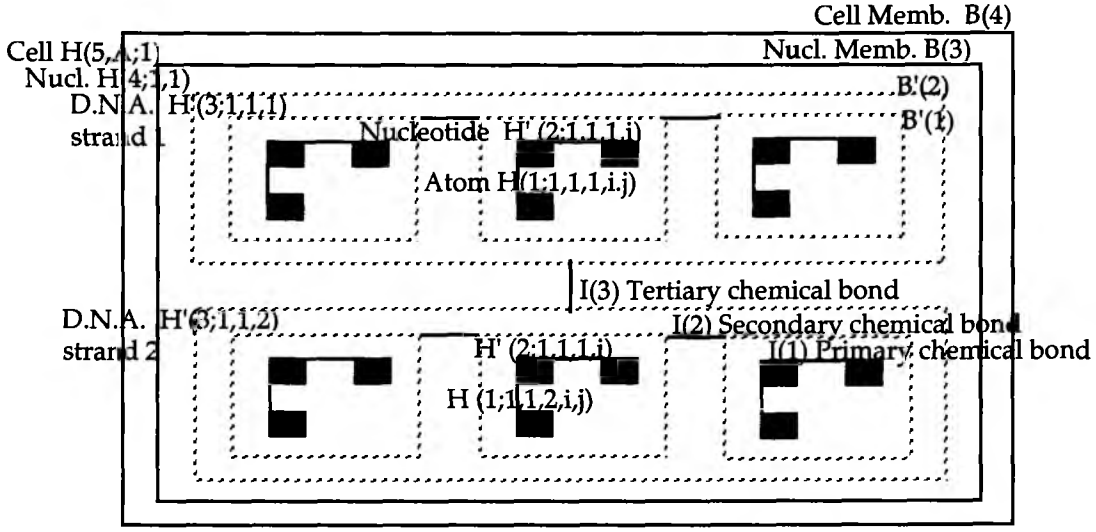
3) Replication of a Cell

Now we will look at replication in a cell. Consider again Figure 43. If elements in H(l) split or replicate, then we have R(l)H(l), otherwise R'(l)H(l). Atoms don't replicate, hence R'(1)H(1). Simple molecules don't split or replicate, so R'(2)H(2). Some macromolecules replicate (DNA), while others do not, denoted by [R(3) ∪ R'(3)] ∩ H'(3). Some organelles replicate (the nucleus), while others do not, thus [R(4) ∪ R'(4)] ∩ H(4). We can now say that the hierarchical equation for replication in the cell is given by Equations 26 below.

$$
\begin{array}{lll}
H(5,S) & = [B(4) &] \cap [I(4) \cup I'(4)] \cap [R(4) \cup R'(4)] \cap [H(4) \cup H'(4)] \\
H(4) \cup H'(4) & = [B(3) \cup B'(3)] \cap [I(3) \cup I'(3)] \cap [R(3) \cup R'(3)] \cap [& H'(3)] \\
H'(3) & = [B'(2) &] \cap [I(2) \cup I'(2)] \cap [\quad R'(2)] \cap [\quad H'(2)] \\
H'(2) & = [B'(1) &] \cap [I(1) \cup I'(1)] \cap [\quad R'(1)] \cap [H(1) \quad] \\
H(1) & = [H(1) &]
\end{array}
$$

where $I(l+1) \cap B'(l) \cap I'(l)$ is empty

Equations 26.



H' (2;1,1,1,i) = Any nucleotide i. i = 1,2,...,n(2;1)
H(1;1,1,1,i,j) = Any atom j in nucleotide i. j = 1,2,...,n(1;i)

Figure 44.

It would be of interest to multiply Equation 22 out, discard the disallowed, and arrive at the subset as the sum of products, and the alternate set plan for this replicating cell. Instead, however, I would like to write an algebraic representation of a cell replicating. To do this, I will first draw the element plan of a cell containing only a nucleus and the two strands of a DNA molecule, each of which is a macromolecule. This is shown in Figure 44. The labeling is shown below and is similar to that of Figure 43, which can be decoded to represent DNA. It will be noticed that, in addition to I(1), the primary chemical bonds between atoms, and I(2), the secondary bonds between molecules, we now have I(3), tertiary bonds between the two DNA strands. Also the labeling of the elements is a little different. The mother cell is H(5,A;1), where A represents the particular subset and 1, this cell. H(4;1,1) represents the nucleus in this cell, and so on, as indicated. B(4) and B(3) are the cell wall and nuclear membrane as in the past.

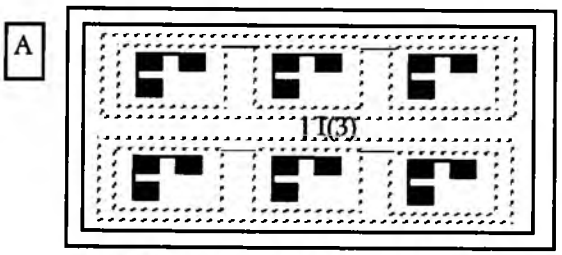
Using this element plan we find the element equation to be.

$$H(5,A;1)= [B(4) \quad] \cap [B(3) \quad] \cap [I(3) \cup I'(3)] \cap [\{B'(2;1,1,1)\} \cup \{B'(2;1,1,2)\}] \cap [I(2) \quad] \cap \{B'(1;1,1,1,i)\} \quad \cap [I(1) \quad] \cap \{H(1;1,1,1,i,j)\}$$

where i=1,2,...,n(2)
and j=1,2,...,n(1;i)

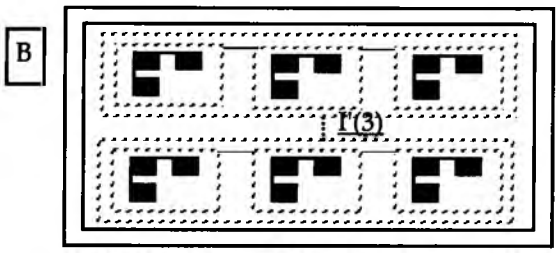
Equation 27.

In Figures 45 and 46, I have shown element plans and equations of a rather simplified cell replication. In Figure 45a (subset **A**) and Equation 28a I have reproduced Figure 44 and its Equation 27. As I(3) becomes uncoupled, going to I'(3), the two DNA strands untwist and we go to Figure 45b (subset **B**), its plan and equation 28b. I have underlined that which has changed.



1 cell, 1 nucleus, DNA strands coupled

Figure 45a.



1 cell, 1 nucleus, DNA strands uncoupled

Figure 45b.

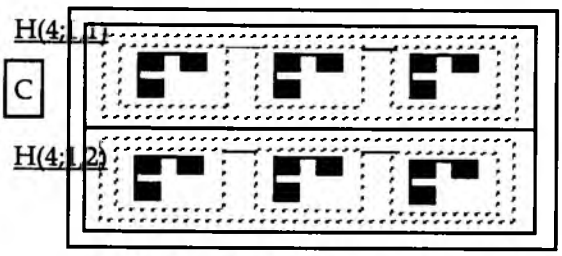
$$H(5,A;1)= B(4) \cap B(3) \cap I(3) \cap \{B'(2;1,1,1),B'(2;1,1,2)\} \cap I(2) \cap \{B'(1;1,1,1,i)\} \cap \{H(1;1,1,1,i,j)\}$$

Equation 28a.

$$H(5,B;1)= B(4) \cap B(3) \cap I'(3) \cap \{B'(2;1,1,1),B'(2;1,1,2)\} \cap I(2) \cap \{B'(1;1,1,1,i)\} \cap \{H(1;1,1,1,i,j)\}$$

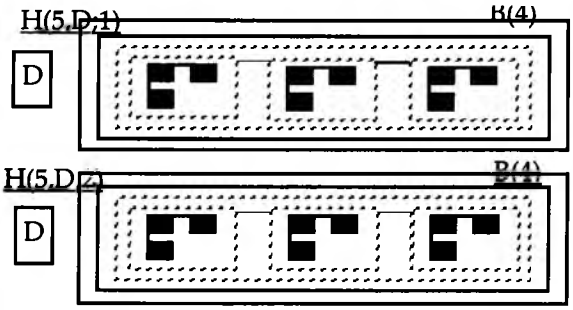
Equation 28b.

where i=1,2,...n(2) and j=1,2,...n(1;i).



1 cell, 2 nuclei, DNA strands uncoupled

Figure 46a.



2 cells, 2 nuclei, DNA strands uncoupled

Figure 46b.

H(5,C;1)=
B(4) ∩ {B(3;1,1),B(3;1.2)} ∩ I'(3) ∩
{B'(2;1,1,1),B'(2;1,1,2,} ∩ I(2) ∩
{B'(1;1,1,1,i) ∩ H(1;1,1,1,i,j)}

Equation 29a.

H(5,D;1) ∪ H(5,D;2) =
B(4) ∩ B(3) ∩ I'(3) ∩
B'(2) ∩ I(2) ∩
{B'(1;1,1,1,i)} ∩ {H(1;1,1,1,i,j)}

Equation 29b.

where i=1,2,...,n(2) and j=1,2,...,n(1;i)
and B(4) = {B(4;1,1), B(4;1,2)

In Figure 46a,(subset C), a membrane B(3) has formed around the new nucleus H(4;1,2) and has retained the membrane B(3) around the mother nucleus H(4;1,1), Equation 29a. Finally a new cell wall B(4) forms around the daughter cell H(5,D;2), and we have two cells, Figure 46b. and Equation 29b. Thus, cell replication moves from subset A through subset D. Although in reality this is a continuous process, I have had to show it as a series of moving plans (pictures) and equations. I have not shown the new DNA strands that form between subsets B and C, primarily for space reasons.

In summary, using the cell space equation and plan as a framework, I have inserted (encoded) into them the growth and replication properties, arriving at the hierarchical equation for both of them. This gave me a vertical look (down through the various levels) at cellular growth and replication. It also showed that replication at the cellular level could be described (simplistically) using a sequence of plans and equations.

Now that the instrument is developed, obviously one can do the same kind of investigation of many other living systems with many other properties. In the next section I will do this with another living system and another property. I hope that the reader can begin to see how this means of investigation might allow one to bring a certain order to living systems.

4) A Hierarical Description of the Systemic Arterial Circulation

a) Diagram of a truncated mammalian systemic arterial system

I would like to describe the whole mammalian systemic arterial system using hierarchy algebra. To make this more manageable, I first simplify by drawing a truncated diagram of the system, turning my attention to only three organs, the vascular remains of the brain, stomach, and kidney. We have then what I have shown in Figure 47. We have the major arteries that lead to these organs, and then the intra-organ arteries and arterioles-capillaries within the organs. For completeness (but not included in the algebraic description), I have shown the venous return.

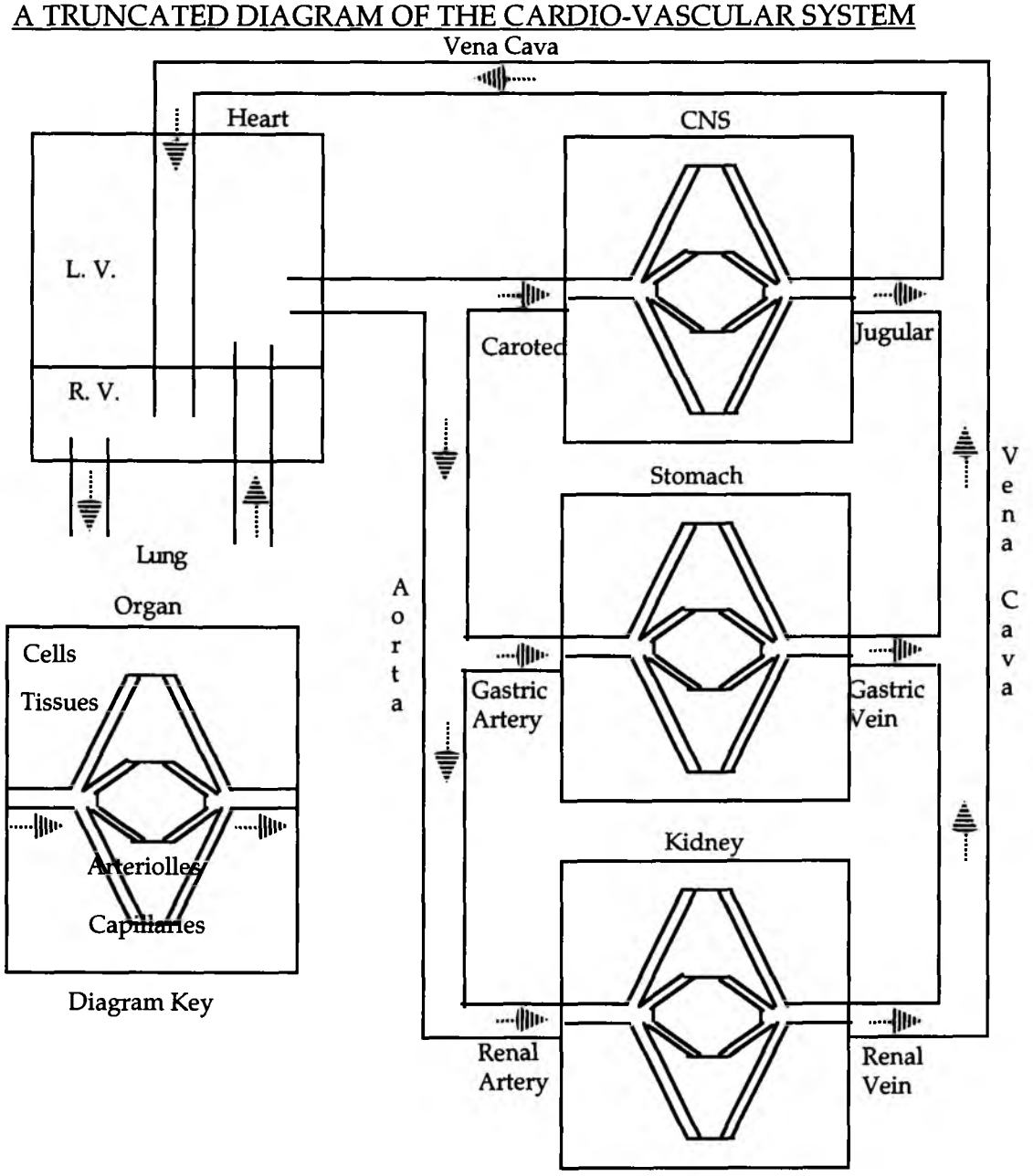


Figure 47.

b) The general model of conduit systems

To describe this system using hierarchy algebra, we start with the generalized equation for any mammalian system (see Equation 11). Then, in a series of steps, we become progressively more concrete, finally arriving at our hierarchical expressions. During our passage we will meet some interesting other interpretations.

The systemic arterial system is part of an organ system, which in mammals has the property of conduits. Let us therefore turn to the organismal system and take the first four levels, the cell being our primitive. The general equation with interactions $I(l) \cup I'(l)$ ignored and the conduit property $C(l,l+1) \cup C'(l,l+1)$ inserted is

$$H(4) \cup H'(4) = \bigcap_{l=1}^{3} \{[B(l) \cup B'(l)] \cap [C(l,l+1) \cup C'(l,l+1)] \cap [H(l) \cup H'(l)]\}$$

Equation 30.

The first expansion gives us the following equations and subsets.

$$\begin{aligned}
H(4) \cup \mathbf{H'(4)} &= [B(3) \cup \mathbf{B'(3)}] \cap \mathbf{[C(3,4) \cup C'(3,4)]} \cap [\mathbf{H(3)} \cup H'(3)] \\
\mathbf{H(3)} \cup H'(3) &= [\mathbf{B(2)} \cup B'(2)] \cap [\mathbf{C(2,3)} \cup C'(2,3)] \cap \mathbf{[H(2) \cup H'(2)\ ]} \\
\mathbf{H(2) \cup H'(2)} &= \mathbf{[B(1) \cup B'(1)]} \cap \mathbf{[C(1,2) \cup C'(1,2)]} \cap \mathbf{[H(1)\ \ \ \ \ \ \ \ \ \ ]} \\
\mathbf{H(1)} \ \ \ \ \ \ &= \mathbf{[H(1)\ \ \ \ \ \ \ \ ]}
\end{aligned}$$

where $C(l,l+1)$ refers to $H(l)$.

Equations 31.

Most organ systems have no boundaries; and so we have no B(3), thus no H(4). Furthermore, all organs have boundaries, so no B'(2) or H'(3). Some tissues have boundaries and some do not; so we have $B(1) \cup B'(1)$.

Matter, energy, and information (MEI) must get from outside the organism down to every cell, and to do this it must pass through the various levels and boundaries of the hierarchical system. MEI is carried by what I have called the $I(l,l+1)$ interlevel or vertical interactions (interactons) and usually this is through vertical conduits or $C(l,l+1)$. The degraded MEI must then work its way back to the outside and this is also via conduits. These conduits occur in various organ systems. They pass through the organ system level C(3,4) and organ level C(2,3), but in some tissues C(1,2) conduits may not occur. That is, I(1,2) MEI may diffuse through the tissue with or without a conduit $C(1,2) \cup C'(1,2)$. Thus our equations now become

$$\begin{aligned}
H'(4) &= [\ \ \ \ \ \ \ \ B'(3)] \cap [C(3,4)\ \ \ \ \ ] \cap [H(3)\ \ \ \ \ ] \\
H(3)\ \ \ &= [B(2)\ \ \ \ \ \ \ \ \ ] \cap [C(2,3)\ \ \ \ \ ] \cap [H(2) \cup H'(2)] \\
H(2) \cup H'(2) &= [B(1) \cup B'(1)] \cap [C(1,2) \cup C'(1,2)] \cap [H(1)\ \ \ \ \ \ ] \\
H(1)\ \ \ &= [H(1)\ \ \ \ \ \ \ \ ]
\end{aligned}$$

where $C(l,l+1)$ refers only to $H(l)$.

Equations 32.

If we let H'(4;i) be any organ system, H(3;i,j) be any organ in this system, H(2;i,j,k) any tissue in this organ, and H(1;i,j,k,l) any cell in this tissue, we can then write the element plan for any conduits in organ systems, and I have done this in Figure 48.

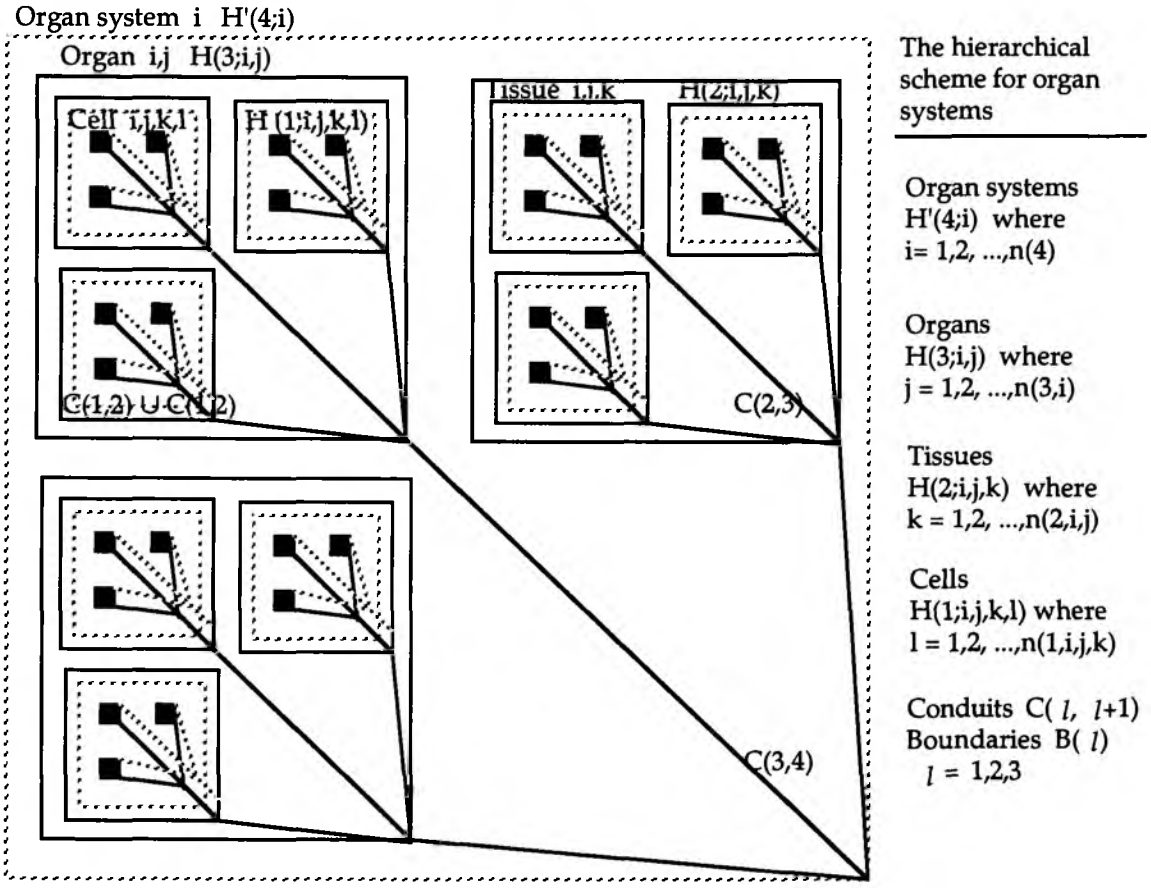


Figure 48.

Since this is very reminiscent of a tree, I have called this a conduit tree. In the equation and its figure it will be noted that there are three set variables, C(1,2) ∪ C'(1,2), B(1) ∪ B'(1), and B(3) ∪ B'(3). There are also three constants, C(2,3), C(3,4) and B(2). This allows 2^3 or 8 terms and 2^8 or 256 subplans or subsets to chose from. There are ten or so organ systems and each one may have several conduit trees in them. A conduit tree may be described using one subset or subplan. With 256 possibilities we have plenty to choose from .

c) Conduit trees in various organ systems

We will consider the mammal. Before turning to the arterial conduit system, however, let us see what other conduit trees we can find within the various organ systems. In Table VI, I have listed some (See Appendix C for more on the nomenclature of mammalian anatomy.). On the far left of Table VI, I have enumerated and written some organ systems H'(4;i) where i = 1a, 1b, 1c, 1d, 3bm, 3bs, 5, 6a, 6b, and 7. The numbers are the organ systems, the letters show when more than one tree is in an organ system. Above the table, I have shown the conduit segment levels C(1,2), C(2,3), and C(3,4) with their interpretation, i.e., what levels they are traveling through. The arrows show the direction of flow of the interactions I(l,l+1).

		Conduits from A — to → B, or A ← from — B		
i	Conduit trees in organ system H'(4,S:i)	cell ↔ tissue C(1,2)	tissue ↔ organ C(2,3)	organ ↔ org. - sys. C(3,4)
1a	Cardio-Vasc. Systemic Arterial Conduit tree	capill- ← arter- aries ioles	arter- ← small ioles arter.	small ← large arter. arter. heart
1b	Cardio-Vasc. Pulmonary Arterial Conduit tree	capill- ← arter- aries ioles	arter- ← small ioles arter.	small ← large arter. arter. heart
1c	Cardio-Vasc. Systemic Venus conduit tree	capill- → ven- aries ules	ven- → small ules veins	small → large veins veins heart
1d	Cardio-Vasc. Pulmonary Venus conduit tree	capill- → ven- aries ules	ven- → small ules veins	small → large veins veins heart
3b m	Nervous system Peripheral motor conduit tree	single motor ← motor fiber bundle	small motor ← motor bundles bundle	large motor motor ← perip bundles n. s.
3b s	Nervous system Peripheral sensory conduit tree	single sensory → sensor fiber bundle	small sensory → sensor bundle bundle	large sens- sensory → perip bundle n. s.
5	Respiratory system Bronchial conduit tree	alveo- ↔ bronch- lus iole	bronch- ↔ bron- iole chus	bron- ↔ upper chus resp. T
6a	Gastro-Intest. Upper G. I. (absorptive) conduit tree	pores ← cry- pts	cry- ← intest- pts ines	intest- ← upper ines G. I. T.
6b	Gastro-Intest. Lower G. I. (excretory) conduit tree	bile canal- → bile ducts iculi	bile → intest- ducts ines	intest- → upper ines G. I. T.
7	Urinary syst. Urinary conduit tree	glomer- → tub- ulus ules	tub- → pelvis ules ureter	pelvis → urine ureter tract

Table VI.

In the cardiovascular system (1a), blood flows through the systemic arterial tree carrying nutrients, oxygen, hormones, etc., from the heart through the large and small arteries and capillaries down to the tissues and cells. On the way back through the systemic venous (1c) tree, it carries oxygen and poor hemoglobin, as well as waste products and perhaps newly excreted hormones, etc. This blood is then carried from the heart via the pulmonary arterial tree (1b) to the

lungs, where it is reoxygenated and returned to the heart via the pulmonary venous tree (1d). It should be noted that C(1,2) or capillaries do not attach directly onto the cells, as is noted in the element plan.

Both the motor (3bm) and the sensory (3bs) divisions of the peripheral nervous system can be looked upon as conduit trees. The large motor nerves carrying impulses begin at the spinal cord and course through organs and tissues, breaking up as they go, to reach the cells. The sensory system begins in sensing cells and works back via single fibers collecting into small bundles and finally large bundles to connect with the spinal cord.

In the respiratory system (5), air moves down through the nose, nasal pharynx, trachea, bronchi, and bronchioles, to the alveoli, being oxygen rich and carbon dioxide poor. Giving off oxygen and receiving carbon dioxide, it travels back the same conduit.

While the preceding conduits pairs might be considered to be in parallel, the gastrointestinal conduit system is in series. That is, the absorptive tree (6a) begins with the mouth, esophagus, stomach, small intestine, and finally descends to the crypts and endothelial cells. As the G.I. tract continues, it carries undigested matter as well as bile (6b), which begins in the bile canaliculi and joins with others to form bile ducts and finally the common duct, which then joins with the intestine. The final conduit tree (7) is mainly excretory and begins in the glomeruli, passes through the tubules, collects in the kidney pelvis, and then out through the ureter, bladder, and urethra.

There are some organ systems that do not seem to have any conduit trees. The endocrine H'(4,S;2) and hemotological H'(4,S;4) systems are concerned with what travels in the conduits (interactons) rather than with the conduits themselves. The cutaneous system H(4,S;9) is a boundary B(4), while the musculoskeletal H'(4,S;8) is concerned with movement M(*l*) and support S(*l*), two properties only briefly touched on before.

d) The hierarchy model of the mammalian systemic arterial system

The system that we are seeking to describe is 1a, the systemic arterial system. Using the equation, the conduit plan, and Appendix C, I can write the set and element equation, and the element floor and elevation plans. Using the set Equations 32, we can change to element Equations 33.

$$\{H'(4;i)\} = [\ \{B'(3;i)\}] \cap [C(3,4)\ ] \cap \{H(3;i,j)\}$$
$$\{H(3;i,j)\} = [\{B(2;i,j)\}\ ] \cap [C(2,3)\ ] \cap \{H(2;i,j,k)\} \cup \{H'(2;i,j,k,)\}$$
$$\{H(2;i,j,k)\} \cup \{H'(2;i,j,k)\} = [\{B(1;i,j,k)\} \cup \{B'(1;i,j,k)\}] \cap [C(1,2) \cup C'(1,2)] \cap \{H(1;i,j,k,l)\}$$
$$\{H(1;i,j,k,l)\} = [\{H(1;i,j,k,l)\}\ ]$$

where i = 1,2,...n(4) ∪ n(4)'
and j = 1,2,...n(3;i) ∪ n(3;i)'
and k = 1,2,...n(2;i,j) ∪ n(2;i,j)'
and l = 1,2,...n(1;i,j,k) ∪ n(1;i,j,k)'

Equations 33.

For the truncated circulatory system shown in Equations 33 and Figure 44 ,we encode the indices as follows (chosen from Appendix C).

i = 3; 6a; and 7; (the three organ systems involved)
i,j = 3,1; 6a,4; and 7,1; (one organ in each system)
i,j,k = 3,1,1; 3,1,2; 3,1,3; (three tissues in the CNS)
i,j,k = 6a,4,1; 6a,4,2; 6a,4,3; (three tissues in the stomach)
i,j,k = 7,1,1; 7,1,2; 7,1,3; (three tissues in the kidney)
i,j,k,l = where l = 1,2,...n(1;i,j,k) (many cells in each tissue)

Equations 34.

For a non-truncated circulatory system (one that includes all the organ systems, organs, tissues and cells in the organism), we use the same Equations 33, but insert all of the numbers from Appendix 3. This gives us what we set out to describe in the beginning.

e) The element plan and tree of the systemic arterial system

In Figure 49, I have drawn the element plans of the systemic arterial circulation, which are isomorphic to each other and to the equation. Once again, the element floor plan shows the "anatomical" relationships, while the element elevation plan shows the horizontal and vertical hierarchy space relationships.

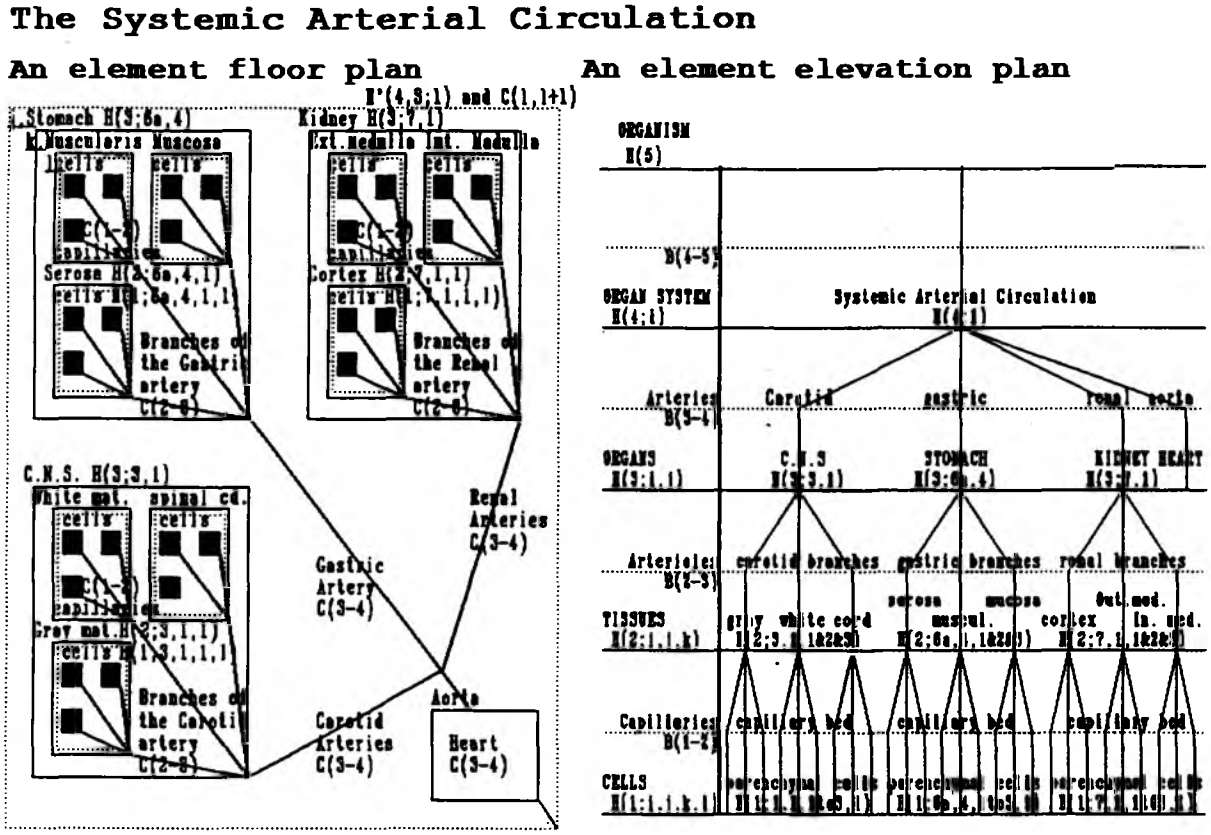


Figure 49.

In summary, I wanted to describe a mammalian systemic arterial system using the hierarchy model. I began with a truncated diagram and an English description of this arterial

system (the data). To describe it using hierarchy algebra, I started with the space equation for all mammalian organ systems, inserted the conduit property, and picked out the particular conduit I was interested in. This was then "instantized" to describe this system, using hierarchical algebra. The description includes an element floor and elevation plan and an element equation. The truncated description can be generalized to describe the entire systemic arterial system. Along the way we came upon a number of other "conduit trees" in the same and in other organ systems. These came as a dividend and demonstrated the power of a deductive system. That is, an abstract expression may contain many concrete manifestations.

5) A Description of the Nervous and Endocrine Conduit Systems

While we are still on conduit systems, I would like to consider in more detail the nervous system and the circulatory system, first, as a means of getting information from the environment to the cell and back, and second, some nervous and endocrine interlevel interactions. This will prove useful later.

To do this, I first cut out of Figure 49 and Equations 31 the two variables B(1) and B(3) and add the top level H(5), to obtain for the organism the stripped-down conduit systems C(l,l+1), where l = 1,2,3,and 4, as seen in Figure 50. Some organ systems H'(4) are in contact through the skin B(4) with the outside of the organism through E(4) ∪ E'(4); for instance, the nervous system with its sense organs. Others are not, such as the circulatory system. All levels of the organism receive MEI through the C(l,l+1) conduits. In particular, they receive Information about the environment through the sensory side C(4-5) of the nervous system, the five senses and their receptors. The organism acts on the environment, on the other hand, through the motor side C(4,5) of the nervous system with muscular behavior.

Organisms have two parallel organ system conduits for feeding information to the organs, tissues, and cells. These are the nervous system's conduit trees and the circulatory system's conduit trees (Figure 50). In the former the interactons are nerve impulses traversing the central nervous system to the peripheral and autonomic nervous systems, where they finally reach the tissues and cells. In the latter the interactons are hormones which travel through the arterial tree to reach the cells. Reaching the cell, they enter the cellular hierarchy to finally influence the macromolecules, molecules, and atoms. I have not shown the cellular vertical interactions in the figure. The nervous system's tree is a rapid alternative to the slower endocrine-circulatory tree.

One should note the fractal structure of Figures 48, 49, and 50. This will be repeatedly seen in the set and element plans of more than one level and are especially noticeable when there are many levels. In the next section, I will draw on the above neuro-endocrine conduit trees as I consider a puzzle that first got me interested in the whole hierarchical problem. Before doing this, however, I want to discuss how different conduit systems interface with one another.

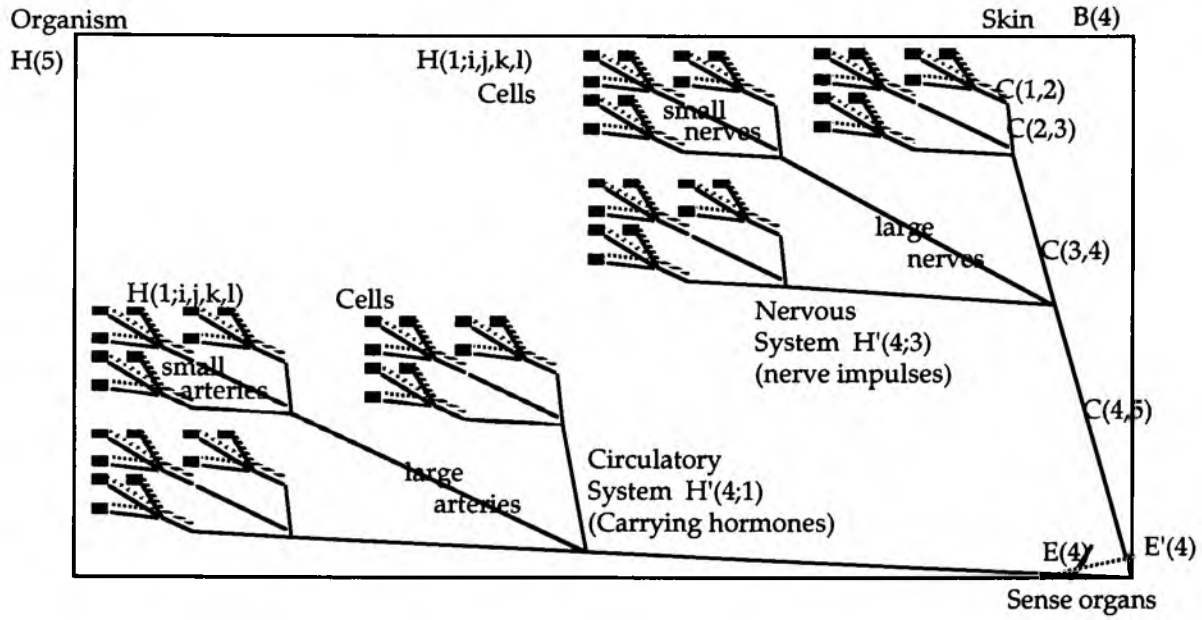


Figure 50.

6) Conduit Interfaces, Convergent and Divergent

Having considered various conduit trees, I would now like to turn to the interfaces between various ones. These may take two forms, which I have shown in Figures 51 and 52 and have called convergent and divergent interfaces. I have changed the shape of the conduit tree for ease of presentation.

a) Convergent interfaces

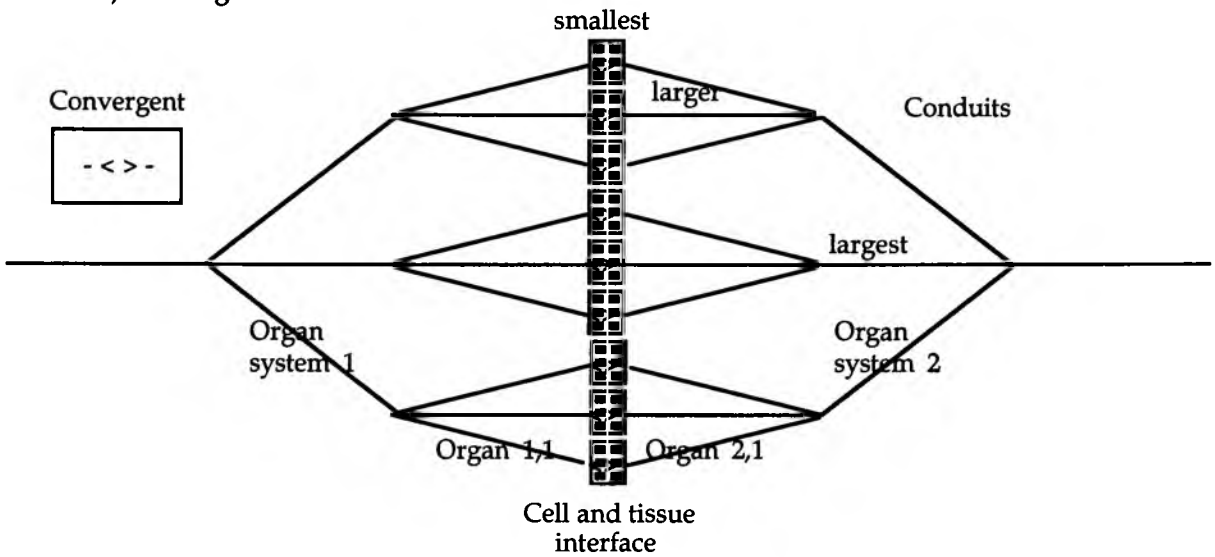


Figure 51.

The first form I have symbolized as -<>- , and the interface between the two organ systems is at the cell and tissue level. A typical example (1 below) is the interface between the systemic arterial and systemic venous systems. In this case, the smallest conduits are the connecting capillaries. Another example (4 below) is between the bronchial tree and the pulmonary artery. In

this case, there is no connecting conduit; the oxygen perfuses through the alveoli into the capillaries. A number of other organ system interfaces of this type are listed below. This list is not meant to be exhaustive. The vertical interactons are usually different in different cases.

One might look upon some organs as structures evolved over time to permit the exchange of MEI at a convergent interface.

1) Systemic arterial	-<>-	Systemic venous
2) Pulmonary arterial	-<>-	Pulmonary venous
3) Portal vein	-<>-	Hepatic vein
4) Bronchial tree	-<>-	Pulmonary venous
5) Upper G.I tract	-<>-	Portal system
6) Portal venous system	-<>-	Biliary system
7) Renal Arterial system	-<>-	Urinary system
8) Peripheral motor system	-<>-	Muscular system

b) Divergent interfaces

The second form of interface, symbolized as >--<, is shown in Figure 52. These are divergent interfaces.

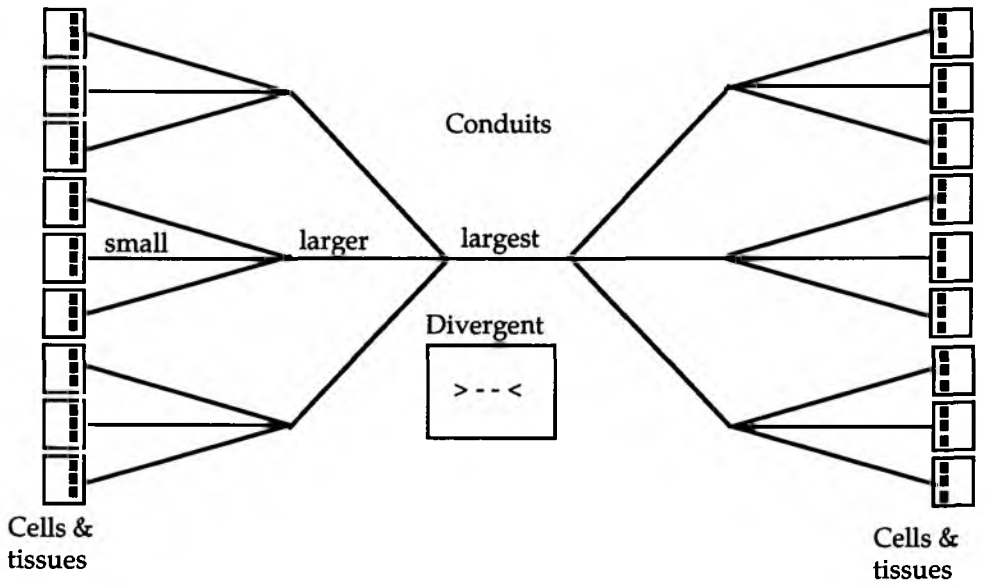


Figure 52.

In these, the conduits are transporting MEI (matter, energy, or information) from one part of a system to another, and this type is mainly confined to the circulatory and nervous systems. Thus, in Example 9 below, we have CO_2 being transported from the tissue capillaries through the veins and heart via the pulmonary artery and arterioles to the alveoli. In Example 11, nutrients are absorbed into the portal capillaries and venules to go by way of the portal vein to the liver, where they are delivered to the hepatic cells. Other examples are shown below. Once again the pertinent interactons are different in the different cases.

9) Systemic venous	>--<	pulmonary arterial
10) Pulmonary venous	>--<	systemic arterial
11) "Distal" portal system	>--<	"Proximal" portal system

12) Central motor system >--< Peripheral motor system
13) Sensory organ systems >--< Central sensory system

In this section we start out, in effect, with the fact that MEI must get from outside the organism down to the cells, and even down to the molecules within the cells. And it must get back again. It does this through a variety of vertical interactons and conduit trees. I also considered some of the interfaces between these trees.

C) CLASSIFICATION OF LIVING SYSTEMS USING THE MODEL

1) A Comparison of the Human and Biological Hierarchical Systems: Levels and their Benchmarks

Before proceeding into the classification of organisms, I would like to review Table I, which I have put in a slightly different format below. Once again, the human and biological systems are compared, there being two different level benchmarks. In one we have a fixed and unchanging benchmark H(0)*; in the other a floating or changing benchmark H(1)*. Each has its advantages and disadvantages. H(1)* is decoded differently in the cellular, human, and social hierarchies.

Level Fixed bm	Level Float. bm	Human systems (hierarchy)	Biological systems (hierarchy)
4	5	Nation	Biogeographic regional
3	4	Communities	Ecosystems
2	3	Ext. families	Populations
1	2	Nuc. families	Families
0*-----	--*1 5 ----	Humans ----------	# 5 level complex organisms -------
-1	4	Organ systems	# 4 level organisms
-2	3	Organs	# 3 level organisms
-3	2	Tissues	# 2 level multicell organisms
-4 -----	---5 1* ---	Cells ----------	# 1 level Protozoa - single cells
-5	4	Organelles	Prokariotes - organelles
-6	3	Macromolecules	Macromolecules
-7	2	Molecules	Molecules
-8	1*	Atoms	Atoms

Table VII.

In the cellular system, the first three levels in both hierarchies are the same. The fourth level, however, may have prokariotes as well as organelles in the biological hierarchy (fourth column), and in the fifth level we have protozoa as well as eukariotic cells.

In the organismal biological system, we are concerned with multicellular, multileveled organisms. If we change our benchmark and make eukariotic cells the first level, we have two-, three-, four-, and, finally, five-level organisms. These correspond to the human levels, cells (1), tissues (2), organs (3), organ systems (4) and finally human (5). These are determined by looking at the data on various organisms regarding their B(l)s and I(l)s (tissues, organs, and organ systems), these defining the levels of each organism. Many of the simpler forms of life, such as jelly fish and flatworms, would be included in the less-than-five-leveled multicellular organisms. This will become clearer after considering Tables 6a and 6b in the next section.

Finally we get to the supra-organismal, societal, or ecological levels of various biological systems, and here I have drawn from Eldredge and Salthe. I have inserted, however, a family level in order to include some of the "social" animals (including humans). For those organisms with a K strategy where a family level can be said to exist, I(1) may be familial or caring interactions,

a K strategy where a family level can be said to exist, I(1) may be familial or caring interactions, while there may be a host of different B(1)s such as territories, nests, burrows, etc. For other organisms there may be no level that we call family (for instance, many fish, insects, or other r strategists). Since, in them, we have I'(1) and B'(1), no H(2) is defined and we can skip immediately to H(3) populations. In the next three levels, populations, ecosystems (communities), and biogeographic regional, while there are B(l) and/or I(l)s defining each level, they are numerous, much vaguer and variable, and more fluid than in the human hierarchy (or seem so to me at least). It should be noted that both human communities and organismal ecosystems may in themselves consist of one or more nested levels. Thus, we might have communities in cities in states, etc., or smaller ecosystems nested in larger ones, etc.. There may well be other levels that could be inserted, particularly in special cases.

2) Classification of Living Systems

As discussed in an earlier section, one can describe hierarchically any living system, using levels and certain properties. One might expect, therefore, that we could use hierarchy algebra to classify living systems. Indeed, the algebra itself has been called class algebra because it divides things into classes. There are several ways one can use the model to classfy organisms.

a) Classification using levels of organization.

When it comes to classifying organisms using levels of organization, it is often useful to place the organism using a fixed benchmark H(0). At other times it is useful to have a floating benchmark H(1). Sometimes one wants to know the number of levels an organism has. Thus prokariocytes are on level H(-5) (fixed) and have four levels of organization. Populations, on the other hand, are on level H(2) (fixed) and have either 11, 7, or 3 levels, depending upon whether you take as the H(1) benchmark, atoms, cells, or organisms.

Let me then place some organisms in a table, grouping them by the level of organization they are on and noting the number of levels they have. I have done this in Tables VIIIa and VIIIb below. On the left, I have labeled the levels using complex organisms as the benchmark H(0). The hierarchical levels are, as usual, defined by boundaries, B(l), and interactions, I(l). Organisms that have no boundary, B'(l), being only defined by interactions, are labeled I $\cap$ H(l). In Table VIIIa, I have chosen several organisms from the "lowest" kingdoms, Monera and Protista. In Table VIIIb, I have chosen several from Animata. I have not represented any organisms from the plant or fungi kingdoms, both because I am not well acquainted with them and because as a physician I am not as concerned with them.

It is seen that in the kingdom Monera we have four, four and a half, and even five-level organisms (the latter not shown). **Bacteria** H(-5) have four levels. **Myxobacteria** pass through stages of three levels, single four-level bacteria H(-5), aggregates of interacting bacteria I $\cap$ H(-5) with four and a half levels, and finally five-level germinating cysts H(-4). I have shown only level four and a half.

In the Protista kingdom we also have several levels. **Protozoa** are single-celled eukaryotes, H(-4), of five levels. All eukariotes have, of course, organelles a level below. **Volvox**, I $\cap$ H(-4), is an organism made up of interacting cells but with no boundary, it thus has five and a half levels. **Algae** ,H(-3), are multicellular, which are all on the same level, but the cells are not only interacting but also boundaried. Thus we can say that they have six levels. Another Protista organism, the cellular slime mold or Acrasiomycota (not shown), may, like its miniature bacterial act-alike, exist on three levels: as an amoeba (five levels), as a migrating slug of many interacting amoeba (five and a half levels), and as a sporophore with surrounding envelope and cap (six levels).

Level	# a c	←MONERA→		←PROTISTA→		
H(-3)	2 6					**ALGAE**
H(-4)	$5^1/_2$				**VOLVOX**	
H(-4)	1 5			**PROTOZOA**	Cells	Cells
H(-5)	$4^1/_2$		**MYXOBACT.**			
H(-5)	4	**BACTERIA**		Organelle	Organelle	Organelle
	$3^1/_2$					
H(-6)	3	Macromol.	Macromol.	Macromol.	Macromol.	Macromol.

Level is with reference to H(0), the complex organism.
is the number of of levels in organism with reference to atoms or cells.

Table VIIIa.

The animal kingdom is made up of multicellular organisms with two, two and a half, three, four, and five levels (the benchmark being the cell). They all have tissues and organs. The only exceptions to this are in the subkingdom Parazoa, which consists of two-level sponges H(-3) (not shown) and two-and-a-half-level **jellyfish** $I \cap H(-3)$. The former do not have tissues, and the latter do not have tissues organized into organs. In the three-level **flatworm** H(-2), tissues, organs, and organ systems seem to melt into one, while in four-level **roundworms** H(-1), organs and organ systems become one. The **coelomates,** it seems to me, are five-level organisms, including **mammals** and **man** H(0). They all have cells, tissues, organs, and organ systems. The difference in these organisms is not in the number of levels but rather in other properties and in the number of elements n(l) on each level. As one descends the evolutionary tree from man down, there are fewer organ systems, fewer organs per organ system on the organ level, and so on down. For instance, in some animals the only organ system is the gastrointestinal system.

Level	# a c	←ANIMATA→				
H(0)	5 9				COELOMATE	MAN
H(-1)	4 8			ROUND WORM	Organ-sys	Organ-sys
H(-2)	3 7		FLATWORM	Organ-sys-organ	Organs	Organs
IH(-3)		JELLEYFISH				
H(-3)	2 6	Tissues	org-tissues	org-tissues	Tissues	Tissues

Level is with reference to H(0), the complex organism.
is the number of of levels in organism with reference to atoms or cells.

Table VIIIb.

b) Using levels together with other taxon-defining properties

Thus far, I have used only two properties, B(l) and I(l) (the level- defining properties), in the classification scheme. Other properties of living systems are obviously critical, and I will now attempt to bring in some of those already defined in the model, as well as some others not yet defined. While many properties (characteristics) are used in defining kingdoms, of prime importance in Fungi, Animalia, and Plantae is the source of matter-energy. That is, animals obtain their matter-energy from ingested food MEi(l), while in plants matter-energy is made MEm(l) using minerals, water, CO_2, and the sun's energy via chlorophyll. Fungi digest food outside themselves by excreting enzymes, then absorbing the products. In this model, mass-energy transfers from one level to another via the I(l,l+1) property. Sometimes this is channeled by the C(l,l+1) property. In these three kingdoms we would expect I(l,l+1) and C(l,l+1) to be quite different. And, in fact, the two organ systems used predominantly in mass-energy distribution in animals (the gastrointestinal and the circulatory systems C(l,l+1) where l=1,2...) are quite different from those used in plants with their leaves,branches, and roots. This was discussed somewhat when we covered organismal conduits.

Another important property in differentiating these two kingdoms is self movement or muscle M(l). Animals have M(l), while plants do not M'(l). A property used in differentiating other classes of animals is the presence of an internal or external supporting skeleton. If we let S(l) represent a skeletal supporting structure on the lth level, and S'(l) be the absence of same, then in this model S(L-1) = B(L-1) when there is an exoskeleton (where L is the top level of the organism). S(l) (where l=1,2,...,L-2) and S'(L-1) would represent an internal skeleton. Reproductive strategies, R(l), are important in classification and are available in the model.

The above discussion of other properties (characteristics) used in aiding classification obviously only scratches the surface of the larger taxi. However, it does suggest that the model might be helpful in systematics. Let me propose a tentative classification scheme using this model.

c) A preliminary classification schema

Starting with some agreed-upon benchmarks, say humans H(0) and atoms H(-8), we could have all those organisms that are made up of interacting macromolecules surrounded by a membrane belong to the kingdom of three levels. These would include the great majority of prokaryotes or Monera H(-5). The four-level kingdom would consist of those organisms having four-level structures, for instance, protozoa, H(-4), which have interacting boundaried organelles as well as cell walls. This kingdom would include most of Protista, except for the algae and some transitional organisms such as the slime mold. One could not use the level properties to primarily classify the other kingdoms because, as seen, they have many levels. Other properties could be used to classify these kingdoms. For instance, one might take the property ingesting matter-energy MEi(l) ∪ MEi'(l), or manufacturing matter MEm(l)∪MEm'(l), or M(l) ∪ M'(l) to help define the animal and plant kingdoms and then work up these kingdoms level by level, further classifying organisms by the number of levels they have. Other properties such as C(l,l+1), R(l), S(l), etc., would be used to subdivide into lower taxa. At the highest levels there would be the higher organisms, including mammals and humans. One might include supra-organismal levels in the classification system.

To summarize the proposed classification system, levels demarcate our main division (kingdoms) for the first two kingdoms (Monera and Protista) and demarcate major divisions (taxa) in the three other kingdoms. Other properties of the model then define the other kingdoms as well as further define our next subdivisions (taxa). These subdivisions or taxa are our subsets and are defined by the other properties that are, or are not, present on each level. Obviously, what the resulting subsets (taxa) are, and how many we have, will depend upon what defining properties (characteristics) we are using to classify the organisms.

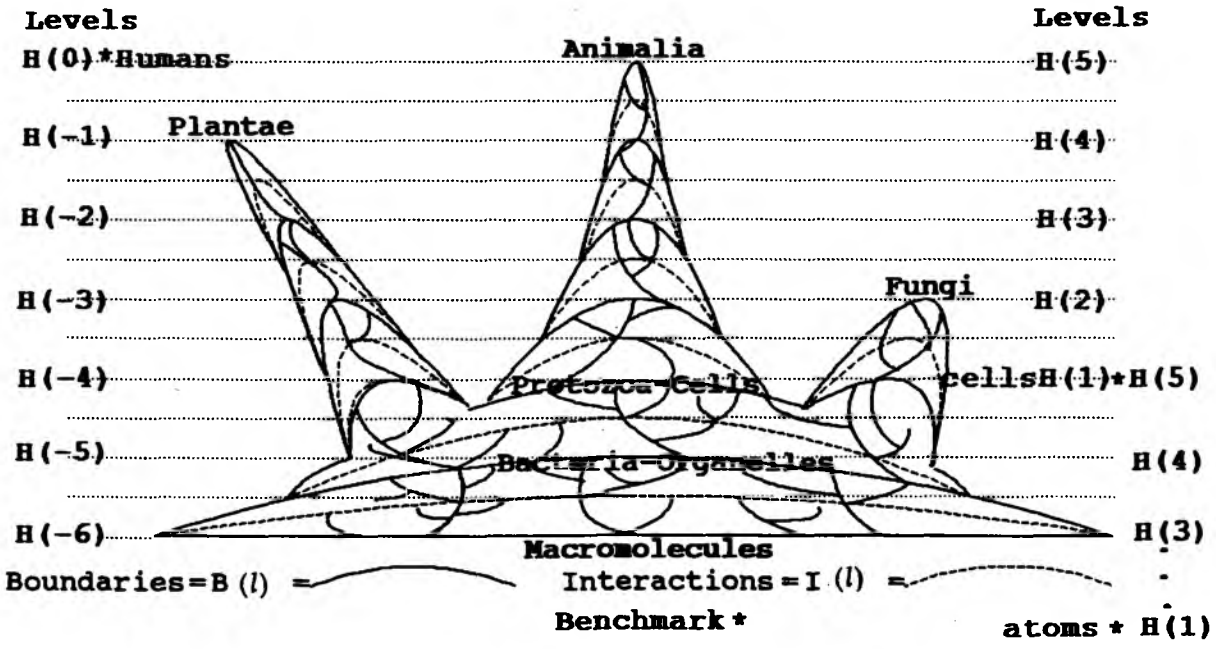


Figure 53.

Perhaps I can make the proposal clearer by showing it in a graphic form. In Figure 53, I have shown on the left the familiar levels with humans as the benchmark, and on the right, the levels with

both cells and atoms as the benchmarks. The figure shows the five newly defined kingdoms. Bacteria H(-5) have four levels; protozoa H(-4) have five levels. Other unspecified properties define the other three kingdoms. The other three are then further divided into lower taxa, as defined by the number of levels they have. I have inserted a fantasized evolutionary tree, mainly to give the gist of the scheme. I strongly suspect that Animalia has only five cellular levels, but I have guessed as to how many levels Plantae and Fungi have. Furthermore, to be more correct, I would have to put "fingers" on each of the three higher kingdoms as their phyla also have levels.

3) Classification of Taxa Using Subsets and the Lattice

I now need to focus on taxa below the kingdom and to relate this to the hierarchy notation: levels, properties, subsets, elements, etc. To do this, I will begin with a particularly simple example. I will present the subset classification first using a table, and then using a lattice. It will be remembered that the lattice can show all of the subsets (taxa) of properties in a particular universe and their relationships. I can picture a huge lattice in which all of the subsets (taxa) of a given level are present, each subset (taxa) being defined by the presence or absence of certain properties (characteristics). Each subset may represent a group of organisms. Although most of these subsets may have always been empty (there never being an organism with these properties), many may be occupied, and many more may once have been occupied by organisms which have become extinct.

a) Classification of a primitive soup

To illustrate, suppose we start with first-level macromolecules in a very primitive "soup." Assuming that these are all on the first and second levels, I would then like to classify (divide into taxa) these first "living" organisms. I will assume the following:

1) H(1) are macromolecules.

2) If one macromolecule can interact I(1)H(1) with other molecules and then split into two macromolecules, then we will say that they can replicate.

3) If they cannot interact I'(1)H(1), then they cannot replicate.

4) The above is much more apt to occur if these molecules are enclosed, crowded together, and "protected" by some boundary B(1). In this case, they become a second-level organism. We can then write the following two-level generating equation.

$$H(2,I) = [B(1) \cup B'(1)][I(1) \cup I'(1)]H(1)$$

where H(1) = Macromolecules; B(1) = Boundary membrane
I(1)H(1) = replicating macros
I'(1)H(1)= non-replicating macros

In other words, we have macromolecules which may or may not interact with other molecules, and when they do, they may replicate. They may or may not be surrounded by a boundary. The universe of subsets is then a way of classifying all of the possible taxa (configurations) that can occur with these properties. This is shown in Table IX. It will be seen that this is similar to the two-level, two-property truth table of Table V. Each of the sixteen subsets (taxa) is shown in five representations. Moving upward on the left there are the algebraic, the binary, the set plan, and the decimal equivalent of the binary. To the right, these representations are decoded to the English representation (interpretation).

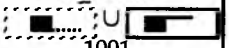
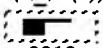
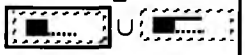
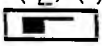
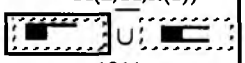
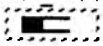
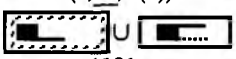
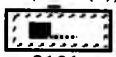
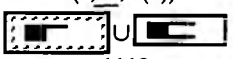
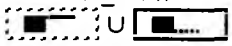
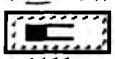
H(2,0;n(1))  0000 [I'(1)∩I(1)] ∪ [B'(1) ∩ B(1)]	undiscribed macromolecules	H(2,9;n(1))  1001 [I'(1)∩ B'(1)] ∪ [I(1)∩ B(1)]	non-replicating and non-enclosed and/or replicating and enclosed micromolecules
H(2,1;n(1))  0001 I'(1) ∩B'(1)	non-replicating and non-enclosed macromolecules	H(2,10;n(1))  1010 I(1)	replicating and enclosed and/or replicating and non-enclosed macromolecules
H(2,2;n(1))  0010 I(1) ∩ B'(1)	replicating and non-enclosed macromolecules	H(2,12;n(1))  1100 I(1)	replicating and enclosed and/or non-replicating and enclosed macromolecules
H(2,4;n(1))  0100 I'(1)∩ B(1)	non-replicating and enclosed macromolecules	H(2,7;n(1))  0111 I'(1) ∪ B'(1)	non-replicating and enclosed and/or non-replicating and non-enclosed and/or replicating and non-enclosed micromolecules
H(2,8;n(1))  1000 I'(1) ∩ B(1)	replicating and enclosed macromolecules	H(2,11;n(1))  1011 I(1) ∪ B'(1)	replicating and enclosed and/or replicating and non- enclosed and/or non-replicating and non-enclosed macromolecules
H(2,3;n(1))  0011 B'(1)	replicating and non-enclosed and/or non-replicating and non-enclosed macromolecules	H(2,13;n(1))  1101 I'(1) ∪ B(1)	non-replicating and enclosed and/or non-replicating and non-enclosed and/or replicating and enclosed macromolecules
H(2,5;n(1))  0101 I'(1)	non-replicating and enclosed and/or non-replicating and non-enclosed macromolecules	H(2,14;n(1))  1110 I(1) ∪ B(1)	replicating and enclosed and/or replicating and non-enclosed and/or non-replicating and enclosed macromolecules
H(2,6;n(1))  0110 [I(1)∩ B'(1)] ∪ [I'(1)∩ B(1)]	replicating and non-enclosed and/or non-replicating and enclosed macromolecules	H(2,15;n(1))  1111 [I'(1) ∪ I(1)] ∩ [B'(1) ∪ B(1)]	replicating and enclosed and/or replicating and non-enclosed and/or non-replicating and enclosed and/or non-replicating and non-enclosed macromolecules

Table IX.

In perusing this classification table, we see several different types of objects:

1) Those that by definition could not be living (e.g., 0000, 0001) but might be necessary precursors.

2) Those that might not be "actively" living but might be a remnant or inactive (e.g., 0100, 0101).

3) Those that are evolving or might have lived at one time but now are extinct (e.g. , 0010, 0011).

4) Those that might still be alive (e.g. , 1000, 1100, 1110).

Obviously, this subset classification is a "bare bones" one, since we are using only two properties. Using more properties would give a much finer classification, but would also require a table and lattice which is too large for this book. The point is, organisms may be first classified by level (the higher taxon), and then each level may be further divided into subsets (lower taxa). To further subdivide and classify the organisms in each subset, one could turn to the number of elements on the second-to-top level in each of the subsets. For instance, in a fifth-level protozoa, which ones

have three types of organelles, which have four, and so on? Or in vertebrates, one could further divide them into the number of organ systems they have, and so on. I have shown this in Table IX in the abbreviated representation that is at the top left of the various representations. H(2;**n(1)**) stands for the number of elements on the second-to-top level. The semicolon (;) separates the set notation from the element notation (as in the past). In this case, of course, we have only two levels, and H(2;**n(1)** refers to the total number of macromolecules on the first level in the system. H(2,2;**n(1)**) refers to the number of replicating but non-enclosed macromolecules, and so on for the various subsets. What we finally end up with is a new number for each of these finer taxons.

For instance, H(3,187;5) is the hierarchical classification number for an organism which has three levels and where the three defining variables (properties) have been found via the truth table to have a binary number of 10111011 (the decimal for this being 187). The binary number in this case is 2^3 , or 8 bits long (one byte). This organism is further classified because i = 5, or there are five elements on the second-to-top level. The more general notation would be H(L,S;n(L-1)), where L is the number of levels of the system as defined by B(*l*) and I(*l*) and S is the subset number (in either decimal or binary form) as defined by the other properties involved. The number of elements on the second-to-top level is i = n(L-1). H(3,I) is the universe of subsets for this three-level, three-property system.

One further point should be reemphasized. When the number of variables involved is over 4, the subset number S may become immense and awkward. In this case, I may frequently just use S for the subset number. For example, in the organism noted below, there are five variables with 2^5, or 32 terms, and H(5,S) would be 32 bits or 4 bytes long.

$$
\begin{aligned}
H(5,S) = {} & [B(4) \quad\quad\quad\quad] \cap [I(4) \cup I'(4)] \cap \\
& [B(3) \cup B'(3)] \cap [I(3) \cup I'(3)] \cap \\
& [B(2) \cup B'(2)] \cap [\quad\quad\quad I'(2)] \cap \\
& [B'(1) \quad\quad\quad\quad] \cap [I(1) \cup I'(1)] \cap \\
& [H(1) \quad]
\end{aligned}
$$

Equation 35.

We can now replace Table IX with a lattice, as I have in Figure 54. All 16 subsets of the universe are present, but I have not interpreted the symbols, as this is awkward and it has already been done in the table. The lattice has the great advantage of showing the meet (intersection) and join (union) and the entailment relationships between the various taxa (subsets). It will be noted that the "layer" number to the left (as defined in the figure) shows the number of terms (products) describing each subset or taxa and also the number of 1s in the binary notation. It will also be noted that with layer 1 we have all the terms (products) displayed that go into making up all of the subsets (as the sum of products). It is seen that properties are added (P'_i --> P_i) as we move to the right and terms are added, 0001 + 0010 = 0011 ...---> 1111 as we move up. Since each term describes one subset of H(1) primitives (in the above case - macromolecules), our living systems (classifications) get increasingly complex as we move up and to the right.

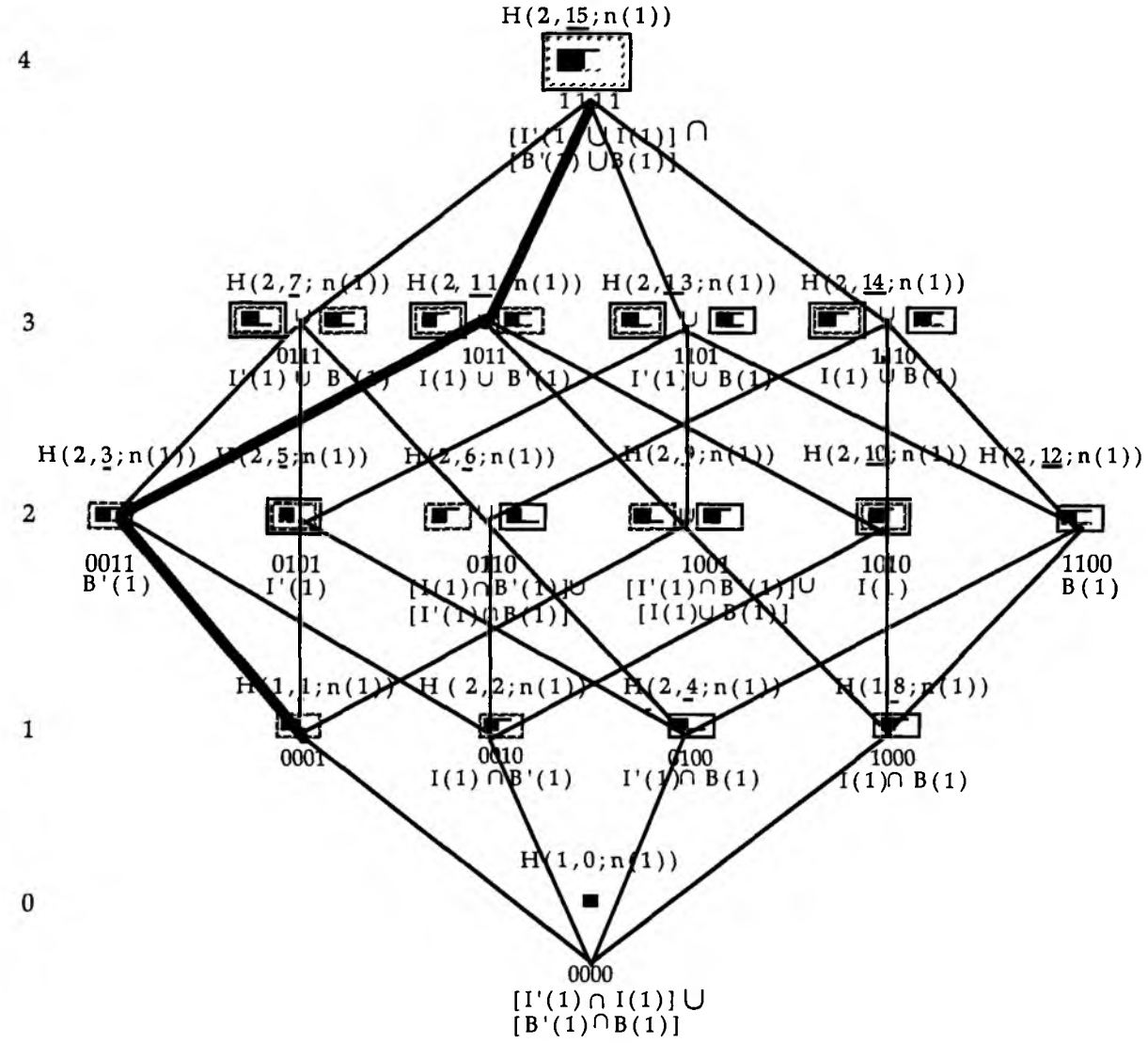


Figure 54.

As we add variables to the description, the lattice increases in size exponentially. That is, for three variables (properties), we have 2^3, or 8 terms, layers, and binary spaces (bits). There are then 2^8, or 256 subsets (taxa). Obviously, this would fill up a whole page, and for >H(3,I) the lattice could not be represented on one page. With these three variables on one level, our binary representation would be 8 bits long and the lattice would have 8 layers. Each level that we add compounds this complexity. Because of the large number of levels that the higher organisms have, one would require the use of a computer to move around the taxa in the lattice. There are some tricks one can use to put it on paper, however, and I will show these below.

b) Using the lattice with constants as well as variables

One may add as many constants as one wishes without increasing the number of subsets, or the size of the lattice. For instance, I may add to Figure 55 any number of constants, say $R(1) \cap I(1,2) \cap C'(1,2) = C$.

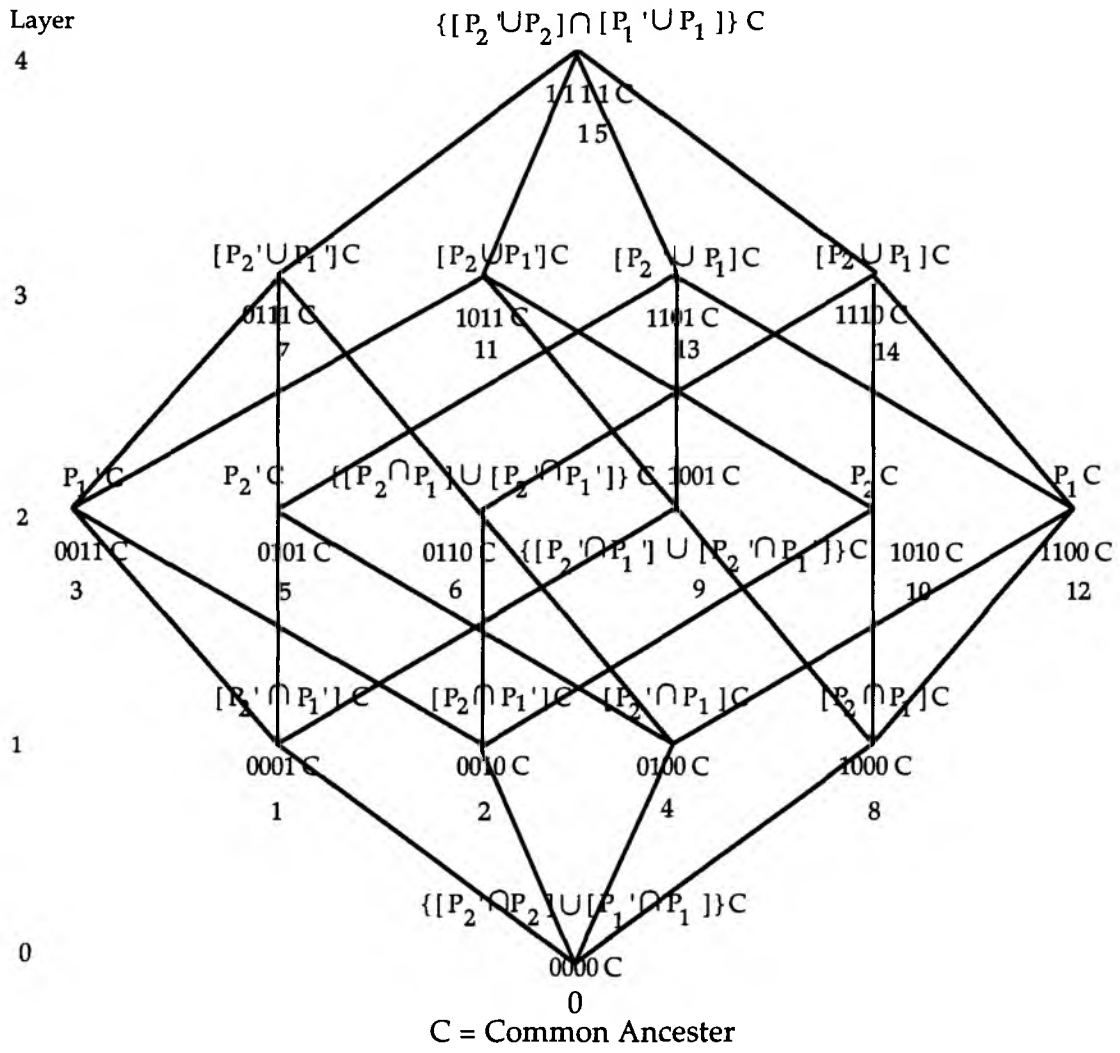


Figure 55.

The lattice size will remain the same as long as I do not add more variables. Using this fact, I have produced Figure 55 above. C is the group of constant properties that a particular organism has, and $P_2' \cup P_2$ and $P_1' \cup P_1$ are two additional properties (variables) that offspring of this "ancestral" organism have. C is constant for all the offspring; they differ only in subsets of properties P_1 and P_2. Thus we can classify all of these organisms using this lattice, since there are 16 different possible subgroups, just as there were with the micro-organisms shown in Figure 54. Once again, not all of the subsets (possible taxa) may be filled; there are empty sets both because of logical and biological impossibility and because of extinction. Although all the organisms in this group have the same C, different groups will have different Cs because they have different properties. If the constants (say C_1 and C_2) for these different groups can be determined and compared, we then have a way of determining what properties they are different in, and thus a way of classifying the higher taxa.

Thus, $C_1 = R(1) \cap I(1,2) \cap C'(1,2)$
and $C_2 = R(1) \cap I(1,2) \cap C(1,2)$

The presence of a conduit between H(1) and H(2) differentiates the higher C_1 and C_2 taxa, while the properties P_1 and P_2 define and differentiate the lower taxa.

D) PREDICTIONS

1) Preamble: Predicting Using Classification Tables

The mendelevium table of chemical elements was long used to predict the presence and properties of undiscovered elements. That is, an unoccupied slot in the table beside its location told you something about an element's mass, valence, and other properties that would be useful in identifying it. Mendeleev himself named one such empty slot eka-silicon (like silicon) and predicted some of its properties. The missing element was found some years later and named Germanium, its properties being quite close to those predicted. Using the periodic table, it was also possible to predict compounds that would have certain properties. Thus Midgley predicted and synthesized tetraethyl lead and Freons. In fact, the periodic and ordered arrangement of the elements suggested that there might be a relationship between this and the structure of atoms, a prediction that led to immensely successful models.

In the same way, the classification of elementary particles using group theory and quantum properties led to the discovery of new elementary particles (the Omega -) and to new models of particle interaction and structure.

I would like to think that the classification scheme that I have proposed, using properties of hierarchical systems, might in a similar fashion be helpful in making predictions in the biological sciences. With this is mind, I have pointed the way, using three evolutionary possibilities.

2) Predictions in Evolution

In the last section I discussed a taxonomy scheme in which biological levels of organization served in defining the highest taxa and in which the next taxa were defined by other properties which were then placed in a lattice. Evolution fits in here quite naturally. Many in the past have noted the intimate relationship between evolution, taxonomy, and levels of organization, and, indeed, one can sense this intuitively. In the present section I will try to make this explicit by describing evolution using hierarchy algebra and lattices. This gives the model a chance to make some predictions.

a) Intralevel evolution - two examples using lattices

It is a very large leap for an organism to evolve to a higher level, and one would expect to find gradations at each level. I would maintain that this gradation of evolutionary steps takes place in what I have called the other properties and subsets of each level. This is best illustrated using two simple examples and the lattice. I refer back to Figure 54 and the example of two-level organisms.

It will be recalled that in this our "organisms" were described using only two properties, boundary B(1) and replication R(1), and I chose for convenience to label R(1) as I(1). Our generating equation then was:

$$H(2,I) = [B(1) \cup B'(1)] \cap [I(1) \cup I'(1)] \cap H(1)$$

Equation 36.

In the highlighted lines of Figure 54, we begin with a non-replicating, non-enclosed macromolecule (0001) on layer 1. To evolve further, it must first be able to replicate. We move to layer 2, position (0011) seeming the most likely. Once it is replicating, for further evolution we need a more constant "internal environment," which necessitates a protecting and collecting boundary (1011) on layer 3. But this does not allow for non-replicating but boundaried macromolecules, so the last step is to 1111 on layer 4. Thus, in this simple scenario, evolution

proceeds upward to complete the first level, layer by layer, along the path indicated, 0001---> 0011---> 1011---> 1111. Another way of expressing this is as the union or join (∪) or sum of the first layer terms. We have, 0001 + 0010 = 0011, 0011 + 1000 = 1011, and 1011 + 0100 = 1111. Other paths might also be possible. For the moment, the important point is that the organism may be said to evolve upward layer by layer. In fact, 1111 or [B(1)∪B'(1)] ∩ [I(1)∪I'(1)] is the join of all or any layer by layer path. Not only does evolution seem to progress upward along these lines, but we can also say the converse, that any organism 1111 has evolved from organisms on lower layers that are on downward connecting lines. For instance, organism 1011 could not have evolved from organism 0110 because it is not on a downward connecting line. Thus this evolutionary lattice has some predictive value. We can predict which evolutionary paths organisms are more likely to evolve along, or which paths they may have already traversed. I say "likely" because more than one path may be possible.

b) The evolution of a gut

Let us consider another example at a different level using other properties. For instance, the evolution of a multicellular organism's contact with the external environment. I start with the two-level generating Equation 37.

$$H(2,I) = B(1) \cap [I(1,2) \cup I'(1,2)] \cap [C(1,2) \cup C'(1,2)] \cap H(1)$$

Equation 37.

I will let H(1) be the set of cells in the multicellular organism.
B(1) is the already evolved boundary around them.
I(1,2) are the cells' evolving interactions with the outside world.
C(1,2) are evolving internal-external conduits.
H(2,S) is the multicellular organism.

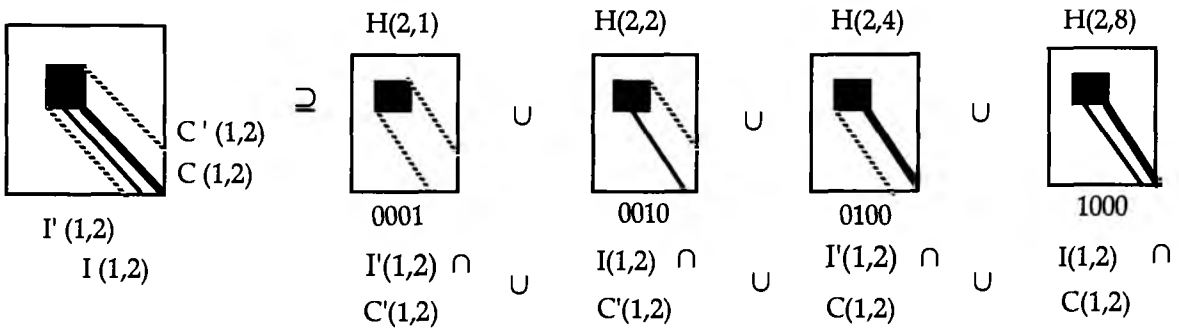


Figure 56.

In Figure 56, on the left I have drawn the generating set plan for Equation 37 and on the right I have draw its four terms. As before, the lines on the right side of H(1) represent C(1,2) and C'(1,2), while the lines on the bottom represent I(1,2) and I'(1,2). The dotted lines are the complements. Once again, the terms are summed in all possible ways to get the universe of subsets. This is shown in Figure 57. It will be noted that the lattice layout is similar to that of Figure 54, except that now instead of I(1) and B(1) we have the new variables I(1,2) and C(1,2). Each subset represents a potential group of primitive metazoans that are evolving into a higher system of interacting with the environment. We begin with highlighted 0010 or H(2,2), an organism in which food diffuses through to the individual cells. In the course of evolution, they acquire an invagination, 0110 or H(2,6). At a later stage, food may enter the organism through the

invagination, 1110 or H(2,14). In other words, the organism has evolved a functioning gut. This further evolves to H(2,15), an organism that can also accept or reject food. Other paths might also be possible, but the point is that the model has acted as a guide, pointed out some possibilities that we might not have thought of, and served as a structure around which data can be organized.

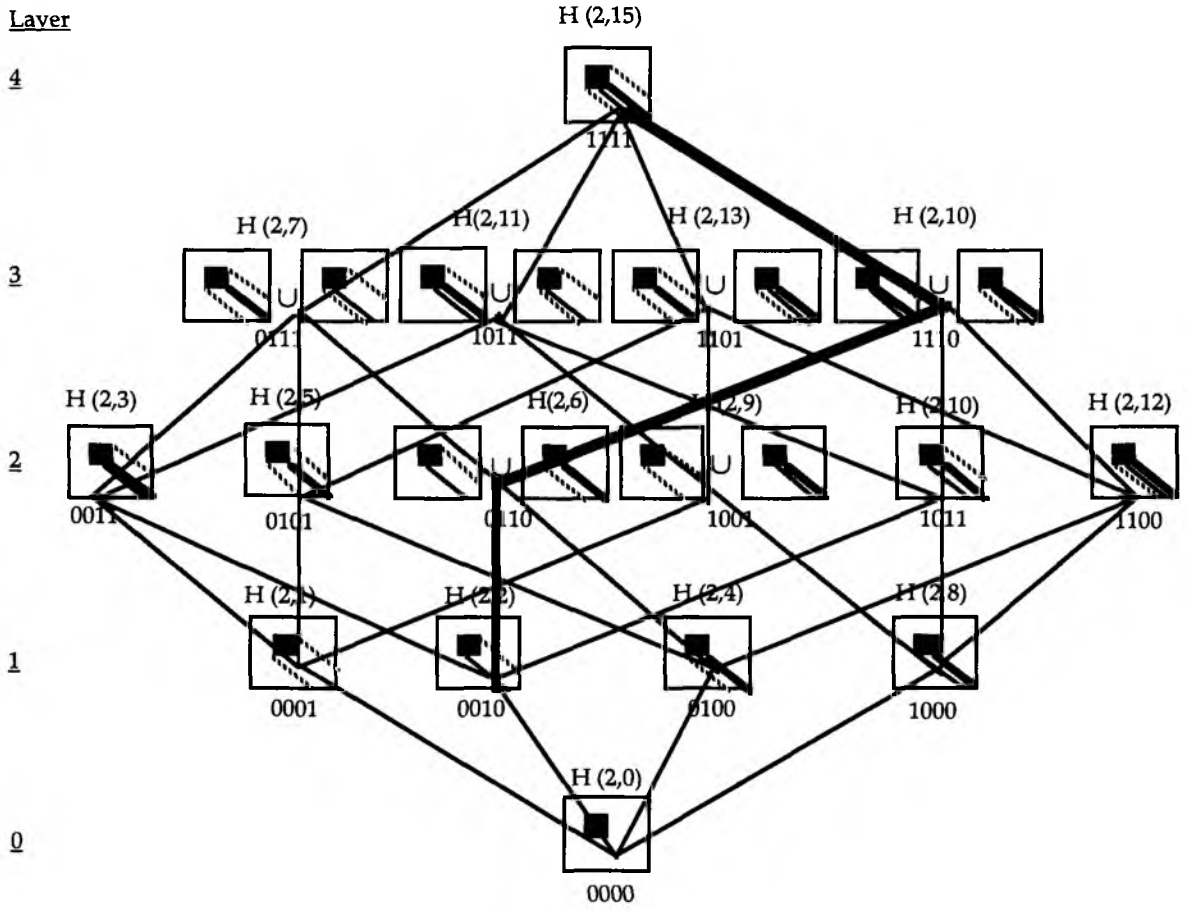


Figure 57.

It should be pointed out that H(1), or the set of cells in this model organism, are primitive and not described. In the real world, of course, these cells have differentiated into a variety of different cells. If we wanted to represent this in our model, we would have to descend several levels, and it would increase the complexity of our model greatly. This could be done, however.

Mathematically speaking, set algebra is a static mathematics. With the above evolutionary interpretations, however, we have a dynamic situation with a living system, H(2,S), changing over time. This is done by considering one system after another (S changing from 2 to 6 to 14, to 15 in the above example), rather like static pictures that can be made dynamic by showing them one after another, as in a movie.

In using hierarchy algebra and the lattice, I have confined myself to two variables, because the three-variable lattice is difficult to represent and any more than three can not represented be on a single page. And yet I suspect that there is much of interest in a multi- propertied lattice, with regard to both classification and evolution. [Editor's note: A computer program could be written that would represent the more complex scenerios.]

c) Species evolution from a common ancestor

Consider another way in which the lattice might be of assistance in thinking about evolution. Let us assume a genera of five-level organisms in which there are five species, one of which is the common ancestor to the other four. Using the five-level hierarchy equation, I have (in fantasy) determined that the equation for the common ancestor is given by Equations 38.

$$
\begin{aligned}
\underline{C} \;=\; & H(5) = B(4) \cap [I(4) \cup I'(4)] \cap G(4) \cap R(4) \cap I(4\text{-}5) \cap C(4\text{-}5) \cap H'(4) \\
& H'(4) = B'(3) \cap I(3) \cap [G(3) \cup G'(3)] \cap R'(3) \cap I(3,4) \cap H(3) \\
& H(3) = B(2) \cap [I(2) \cup I'(2)] \cap G(2) \cap R'(2) \cap I(2,3) \cap C(2,3) \cap H(2) \\
& H(2) = B(1) \cap I(1) \cap [G(1) \cup G'(1)] \cap R'(1) \cap I(1,2) \cap C'(1,2) \cap H(1) \\
& H(1) = H(1)
\end{aligned}
$$

Equations 38.

Since this is a constant for all the species in this genera, I have made it equal to a constant $\underline{C}$. The species differ one from another by how they evolve two additional properties, P_1 and P_2. I have illustrated this in Figure 58.

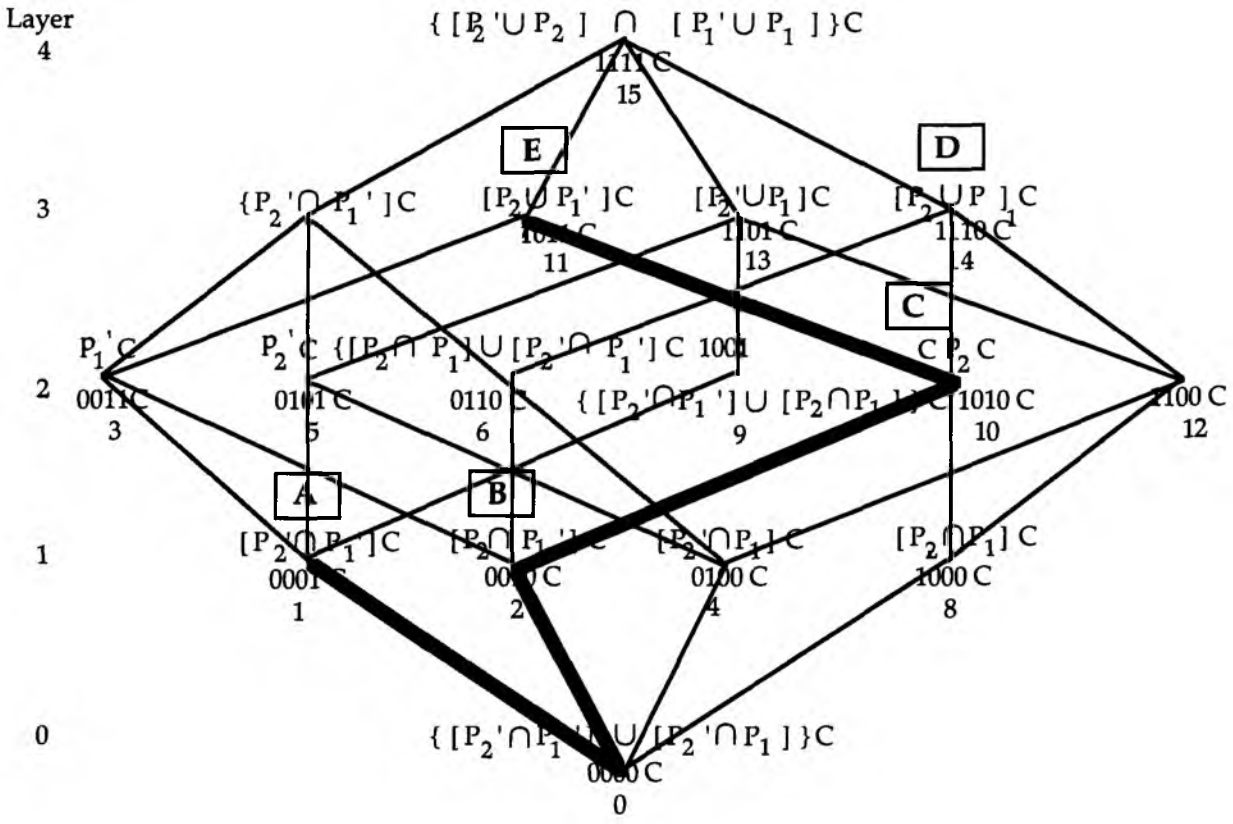


Figure 58.

It will be seen that the common ancestor on the 0 layer 0000 $\underline{C}$ is not described in terms of the two new properties. They are first noticed by their absence as 0001$\underline{C}$, (A), or $[P'_2 \cap P'_1]\underline{C}$. That

is, this is a species without the properties in question. Another species (B), however, has evolved from the common ancestor to the first layer, this one becoming 0010$\underline{C}$ or $[P_2 \cap P_1'] \cap \underline{C}$. It has property 2 but not 1. From (B), a third species (C) evolves to the second layer, position 1010$\underline{C}$, $P_2\underline{C}$. This species has property 2, but property 1 is not mentioned and it might be present. This further evolves to species D on the third layer, it being 1110$\underline{C}$ or $[P_2 \cup P_1]\underline{C}$. This species could now have either property. Or (C) could evolve to species (E), 1011$\underline{C}$ or $[P_2 \cup P_1']\underline{C}$.

This species has the possibility of not having property 1.

d) Evolution and levels of organization

Consider Equation 39 below, which is for any three-level organism, H(1) being macromolecules and there being a number of properties on each level.

$$\{H(3;i)\} = [\{B(2;i)\} \cup \{B'(2;i)\}] \cap [I(2) \cup I'(2)] \cap [P(2) \cup P'(2)] \cap [\{H(2;i,j)\} \cup \{H'(2;i,j)\}]$$
$$\{H(2;i,j)\} \cup \{H'(2;i,j)\} = [\{B(1;i,j)\} \cup \{B'(1;i,j)\}] \cap [I(1) \cup I'(1)] \cap [P(1) \cup P'(1)] \cap \{H(1;i,j,k)\}$$
$$\{H(1;i,j,k)\} = \{H(1;i,j,k)\}$$

where $P(l) \cup P'(l)$ may be multiple.

Equation 39.

It seems to me that there are several ways by which this organism could evolve: 1) By an increase in the number of levels of organization or 2) by an increase in the number of other properties (thus subsets). I am concerned now with the first way, because this model predicts that eventually two more levels would evolve in at least one species of organism. I can think of three ways in which levels might be added:

a) H(3;1) might interact through I(3) with H(3;2) and other elements at this level and when surrounded by a new boundary B(3) become a new organism H(4) at a higher level.

b) A new level might be inserted in between existing levels rather than on top as in a). This would push up the top level H(3) to H(4). This could occur if:

i), another organism with a different B and I, was incorporated into the first, as might occur with symbiotic evolution and presumably did occur with the evolution of eukariotes.

ii) A new B(l) and I(l) evolved from the $P(l) \cup P'(l)$ possibilities (where l = 1 or 2).

The data to see whether these predictions are correct may already be available. A brief word should be said about evolving and embedding properties. Consider the organism represented by Equation 32. As properties are added to either the fourth or fifth levels, we say that these properties are evolving. On the other hand, as we add these levels, properties on the fourth and third levels are being embedded. The two processes are obviously reciprocal. If our focus of interest is on H(l), moving up to H($>l$) shows properties as they evolved, while moving down to H($<l$) reveals embedded properties.

3) Other Predictions

Once one has an encoded formula that can be manipulated, all sorts of predictions are possible. Rather than do this, however, in the next section I will turn to some broader implications and considerations of the model.

E) INFERENCES AND OTHER MATTERS

In this section I will look at some broader implications of hierarchy theory. In addition, I will expand on some ideas not adequately covered in the preceding text and on some thoughts of a more philosophical nature.

1) External and Internal Hierarchical Structure

Thus far in this book, when referring to the hierarchical structure of a system or object, I have usually meant its internal structure. However, it is frequently convenient or necessary to talk about the external structure or environment of a system. A change in notation is helpful when we wish to do this. The system of concern, or subject, has been the hierarchy H(L), where L has been the top level of the system and H(1) has been the primitive building blocks. Let us change this notation, however, and make the element H(0;i,j,...) the subject. Then negative level numbers, l = -1,-2,...,-L represent the internal hierarchical structure of the subject, -L being the elements on the lowest level, the building blocks. The positive level numbers, l = 1,2,...+L represent the external structure or environment of the subject. This may be made clearer by referring to the example shown in Figure 59.

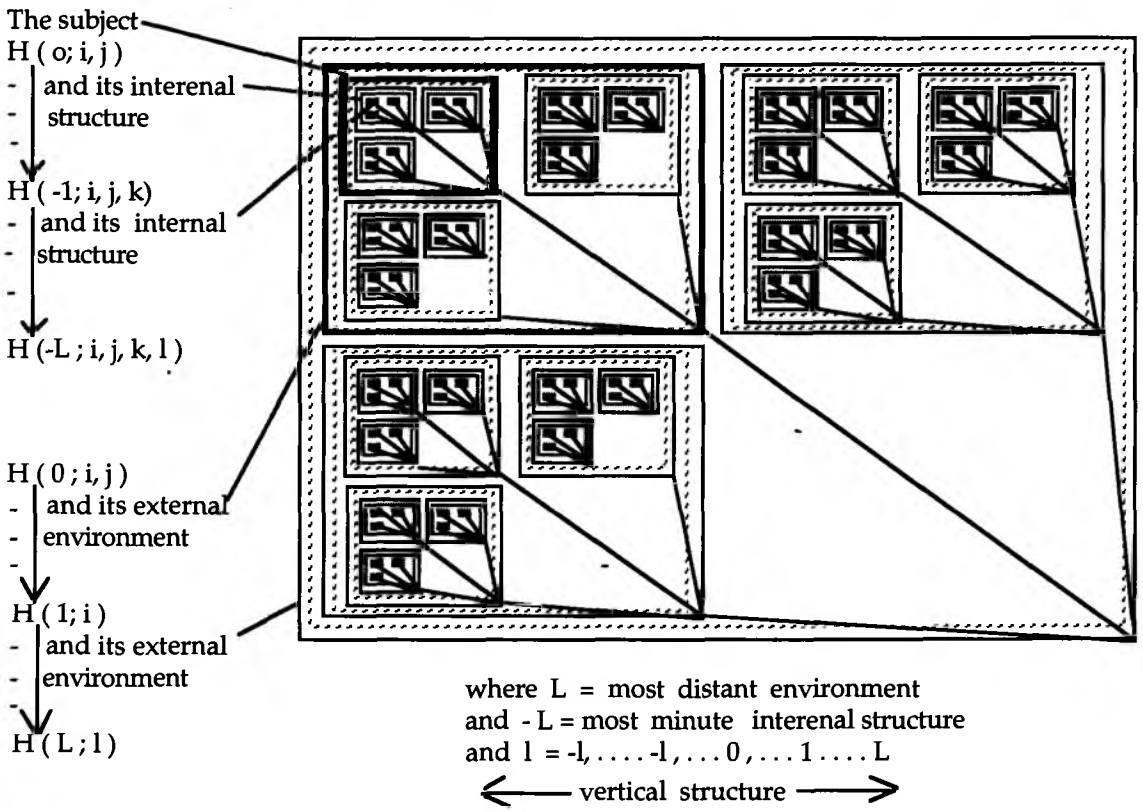


Figure 59.

In the figure, we have a multi-level element plan in which the subject is surrounded by the heavy line. I have labeled this H(0;i,j). The immediate environment of the subject (surrounded by a lighter line) is H(1;i). The further environment, or what I will call the field, is then labeled H(L). The internal hierarchical structure of H(0;i,j) is H(-1;i,j,k), where k = 1,2,.... At the lowest level, we have H(-L;i,j,k,l,...), where l = 1,2,... ,n(-1;i,j). This is, perhaps, seen more clearly in Figure 60, where I have shown the element elevation plan. I have enclosed the subject (an element), again using a heavy line. Its immediate environment is then H(1;i) (a lighter line), excluding H(0;i,j), while the more distant one is H(L), again excluding the same.

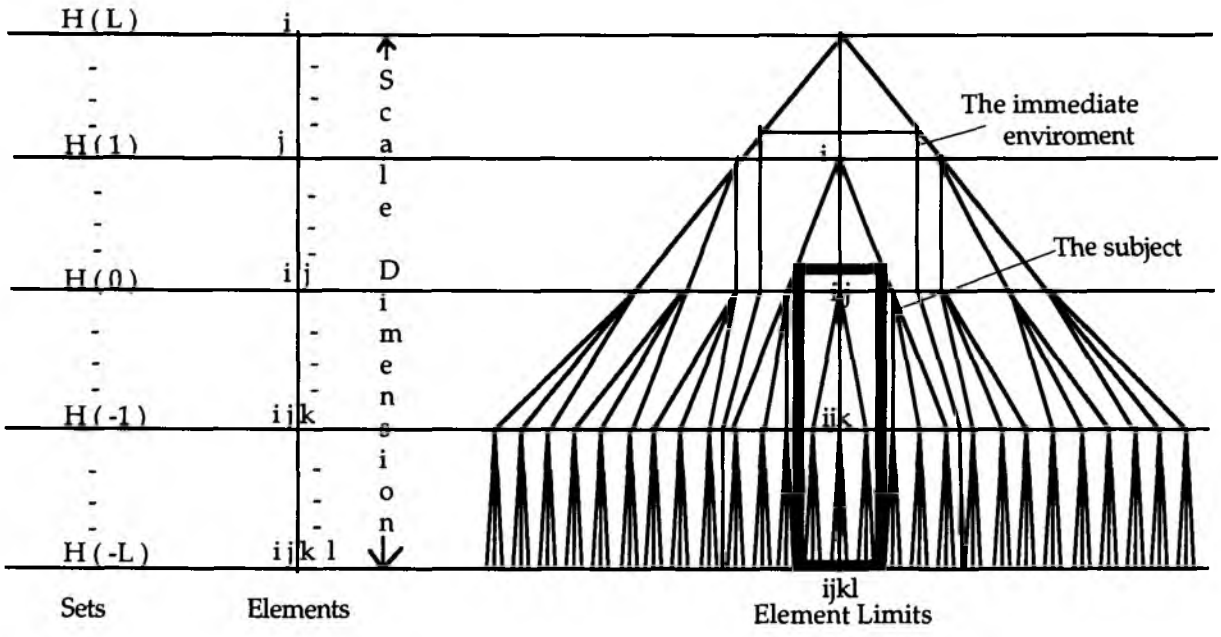


Figure 60.

With this notation system, we can describe as much internal structure and external environment as we wish; we only have to move down, decreasing -l, and up, increasing l, adjusting the indices as we go. In this particular case, I have drawn -L as the most minute structure and +L as the most distant environment. This necessitates many indicies although I have shown only four. Obviously, the number of indices varies with the number of levels.

Let me give a few examples. If our subject H(0;i,j) is a particular human being, then H(1)={H(1;i)}, where i=1,2,... is his nuclear family and H(2) is his extended family. Internally, we have H(-1)={H(-1;i,j,k)}, where k=1,2,... are his organ systems, and H(-2)={H(-2;i,j,k,l)}, where *l*=1,2,... are the organs in organ system k. Another example: if our subject H(0;i,j) is any cell, then H(1;i) is the tissue it is in and H(2,1) is the organ it is in. On the other hand, H(-1;i,j,k) is any of its organelles and H(-2;i,j,k,l) is a macromolecule within this organelle.

It should be pointed out that when I speak of environment, I mean a very special kind, namely, one that has only systems or objects similar to the subject or any of its contents. Thus the environment of an organism in this model consists of other organisms, or their subsystems. Non-organic elements are not represented except at the molecular or atomic level.

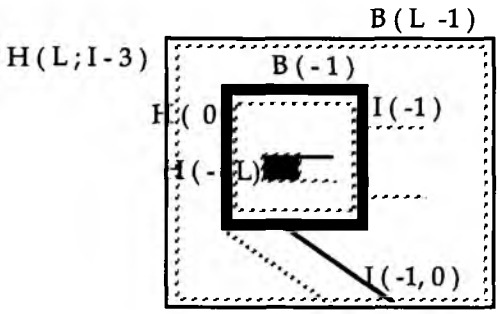
2) The Fractal Nature of Living Systems

It has been mentioned previously that the set plans appeared to be fractals; and, indeed, since fractals can be described as designs having self similarity under repeated magnifications, they are fractals. This can be seen clearly in the element plan of Figure 59. There are other features of set and element plans that are reminiscent of fractals also. Consider the "seed" equation and plan of Figure 61 below. The plan generates a set plan; and when one enumerates this, we wind up with the element plan shown in Figure 59. As in the past, the equation and the plan are isomorphic to one another. It should be noted that simple equations (of a different sort) generate the Mandelbrot fractals.

Fractals have self-**similarity** under magnification. They are not self **Identical.** The plan of Figure 59, except for size, is identical to itself. It generates the universe of subplans and many of them are self-similar. That this is the case can be seen in Figures 27, 32, and 38, which compare the generating plans to the descriptive plans.

$$H(l+1) = \bigcap_{l=-L}^{L-1} \{B(l) \cap I(l) \cap I(l, l+1) \cap H(l)\}$$

Generating equation and seed, multileveled, 3 property hierarchy



Generating set plan and seed of
this multileveled, 3 property hierarchy

where $B(l) = B(l) \cup B'(l)$
and $I(l) = I(l) \cup I'(l)$
and $I(l, l+1) = I(l, l+1) \cup I'(l, l+1)$
and $H(l) = H(l) \cup H'(l)$
and $H(0)$ = the subject
and $1 = -L \ldots -1 \ldots 0 \ldots 1 \ldots L-1$
and $H'(-L) = 0$

Figure 61.

3) Using the Hierarchy Model to Organize Biological Data

One of the most potentially useful applications of this hierarchical model is in organizing the data of the biological sciences. Most biologists intuitively acknowledge the hierarchical structure of living systems; that objects of their study can be arranged on different levels of organization. In this work, however, I have tried to make the implicit explicit. That is, I have tried to explicate how their objects of study and their properties can be arranged hierarchically using set algebra.

In describing a living system using the hierarchy equation, I have frequently said that if one looks at such-and-such system, one sees such-and-such. Obviously, I did not mean actually to look at the system now. Rather, I meant that previous studies have shown that one finds the following data. This, of course, is a very selective process and one that is done in all of the hard sciences. In the method I have employed, I have looked at some of the hierarchies found in the life sciences and from them picked properties that were relevant to many different levels. These properties were then inserted into the hierarchy equation so that we end up with a description which is organized along hierarchical lines. Data which may have come from a number of different fields is now organized in a unifying fashion.

Not only are the objects of study organized hierarchically but so are their properties, attributes, or functions. I have picked properties that span many levels rather than those that are confined to just a few. These, then, are also arranged hierarchically. This reduces the reductionism problem almost to zero. That is, we do not have to complete our understanding of a lower level before we can understand the one above it. Our understanding becomes greater with each new property that we explore vertically. Of course, there are always going to be levels which have unique properties, but these can be considered later in the organizing process.

4) Health and Disease

Some years ago, while working with Dr. George Engel in the psychosomatic unit at the University of Rochester Medical School, I became fascinated with an old observation that I found much more prevalent than previously thought. The observation was that in the setting of a significant personal loss, people could become ill and even die. There is an old saying that "Aunt Jennifer died of a broken heart." Now it was known, of course, that Aunt Jennifer did not actually die because of a ruptured myocardium. During her grief her body was in some way affected and she developed pneumonia or the like. How could this be? In what mysterious ways did the loss of her husband make her lung a more suitable culture medium for a pneumococcus bacillus, or her heart more prone to a sudden arrythmia?

I had the intuitive feeling that the answer to this problem lay somewhere in the hierarchical structure of living systems. But what is hierarchical structure? How do we explicate it so that it might throw some light on our subject? I wanted to make as rigorous an inquiry as possible. Further back in my past, it had been drilled into my head that "the preferred language of science is mathematics" (Princeton University physics department, 1950). Could hierarchical structure be approached mathematically? Once again, intuitively I thought it might be.

I have spent too many years moving from that intuitive feeling to something concrete, something which I hope to communicate in this book. It has been an exciting venture and one that has opened up new avenues and vistas, many of which I came upon quite unexpectedly. In this section, I want to return to Aunt Jennifer and her problem. I certainly hav not solved it, but I hope that I may have put it into new perspective.

The real world for the physician is the world of Sam Smith and Sue Johns, of Nancy and Chris. It is of heart and stomach, of gastric mucosa and parietal cells. One is concerned with HCl and antacids, with atrial pacemakers and electronic pacemakers. And then there is Sam's family and his home. He just lost his job when the community shoe factory closed down. As a practicing physician, one has to cover a lot of ground, both horizontally and vertically. There are a lot of levels to jump.

Health and disease are seldom on just one level. It would appear that disease conditions are similar to areas of disorganization in an otherwise rather well-organized hierarchy. Because of redundancy, both horizontally and vertically, these areas of disorganization may not be immediately manifested as disease. However, if we have large enough areas or several areas of disorganization, either on the same or different levels, and these impinge on one another, we may then get overt disease. But how do these areas impinge on one another? One would suspect they impinge through horizontal interactions if they were on the same level, or vertical interactions if on different levels.

To give an example, in the hemolytic anemia due to glucose-6-phosphate dehydrogenase deficiency, we have a genetic disorganization at the macromolecular level (DNA), which produces horizontally the enzymatic deficiency. This, in turn, gives rise vertically to a cellular predisposition to hemolysis. If a person is exposed to a disorganization at the social level, say a malaria epidemic, and receives the oxidating agent primaquine, this may vertically trigger hemolysis of the predisposed cells. We have two areas of primary disorganization, two necessary but not in themselves sufficient conditions for an illness. Together, however, they may merge and become sufficient.

Another example, this time with Aunt Jennifer: It has been shown that emotional factors can influence the neuro-endocrine-immune system in such a way that the organism may become more susceptible to infections. This is through the neuro-endocrine conduit trees and interactions discussed in the previous section. Thus when Aunt Jennifer's husband dies, there is a disruption at the family level, which affects Aunt Jennifer and her neuro-endocrine system. This is transmitted

down her neuro-endocrine conduit trees to the lymph cells, and further still to the macromolecular antibodies defending her against pneumococcal infection. But if such a bacterium is present, and the antibody production is compromised, then such an infection might arise, and, indeed, she might succumb to it.

Still another example: Some years ago I did some clinical research on patients with adult celiac disease, who were all diagnosed when they were children. The hypothesis I was testing was that there are three necessary, but none by themselves sufficient, conditions that must be present before a patient can develop symptoms of Celiac disease. When all three of them are present, symptoms may develop.

These three conditions were:

1) A genetic factor giving rise to an enzyme deficiency having to do with the breakdown of gluten in the gut. Gluten is the principal protein in bread.

2) The presence of gluten in the diet.

3) A significantly depressive affect, the latter frequently being precipitated by the loss of a key person or object.

Conditions 1 and 2 were known from research already done. However, these conditions were, in my (and others') experience, frequently present while there was no evidence of the disease process. But let the patient become depressed, and then a relapse occurred. My research supported this hypothesis. However, I became more interested in the present work, and the complete study was never published.

If Aunt Jennifer had such a gluten-splitting enzyme deficiency and were to have suffered a relapse with the death of her husband, I would look at it as follows. She had a hereditary defect at the level of the macromolecule DNA, which led horizontally to a decreased or altered gluten-splitting enzyme production. She was able to produce just enough enzyme, however, to split the gluten bread she was eating, until she had a change in organization at the level of the family. This once again traveled vertically down through her neuro-endocrine trees, right down to the existing genetic defect. Enzyme production was further compromised, and she developed the relapse of her Celiac disease. If we were to withdraw bread, she would undoubtedly get better, although she might still be depressed.

The broad picture that I hope has begun to form in the reader's mind is that organisms (humans) are part of an immense four-dimensional network which extends both horizontally and vertically, as well as through time. Any one human is a node in this network, interacting horizontally [I(l)] with other humans and vertically [I(l,l+1)] with other levels, both inside and outside itself. Disturbances in any location in this network may be experienced throughout a significant part of it. A two-dimensional analogy is a large spider web which has experienced the disturbance of an insect landing in it; this disturbance is felt throughout the network. If there is a "weak link" in the web or network, it might break at this "link." Or the weak link may put a strain on other weak links, weakening them further. I have not formally defined either organization or disorganization in this work, and I will not. I hope, however, that the reader can intuitively relate organization to the network or web and disorganization to "weak links."

In most illnesses that physicians deal with, there are multiple causative factors. That is, there are a number of primary areas of disorganization, either on the same level or on different levels. A disturbance in the organization in one area on one level may be experienced throughout. If these other primary areas of disorganization are affected enough by the disturbance, we may get symptoms and what we call illness. Sometimes it may take a number of areas impinging on one another before symptoms occur. Furthermore, what is a disturbance for one person may not be for another. Or what bothers one person in one way may bother another in a different way. A death in the family may precipitate celiac symptoms in some, pneumonia in others, and jubilation in still others.

5) The Hierarchy Model and the Study of Living Systems

Some of the difficulties in investigating living systems is brought out by considering the hierarchy floor plans. Consider the multileveled, three-property element floor plan shown in Figure 59. As discussed in the earlier section, I have changed my notation system so that H(0;i,j) is the subject, <H(0) is the interior of the subject,and >H(0) is its environment. The properties I have drawn on are the boundary around parts B(l)∪B'(l), the intralevel interactions between parts I(l) ∪I'(l), and the interlevel interactions between parts I(l,l+1)∪I'(l,l+1). The five-level figure is extrapolated to levels -L,...-l,...0,...+l,...+L.

In order to investigate the subject, we must invade its distant as well as nearer environments, breaking through boundaries and interactions just to get to it. To investigate the subject's interior, we then break through its boundary B(-1) and look at the parts H(-1), their horizontal interactions I(-1) ∪ I'(-1), as well as their vertical interactions (the interactions between H(-1) and the next subset above). To investigate further the parts, we then must break through their boundaries and repeat the investigative process with each of these smaller parts H(-l). It is obvious that as we investigate the subject, we alter many properties in ways that may change its subset description and nature. However, we have no choice. If we do not investigate in such a fashion, we cannot get at the subject and its interior at all. In investigating living systems, I believe we sometimes forget that in isolating the subject of investigation, we alter its nature even before we start investigating. I believe that this model helps to keep the problem up front.

I was initially a bit surprised to find that nature seemed to be arranged so hierarchically, to indeed find these boundaries, interactions, and levels. But then I realized that this is not really so surprising, for our brain works this way and our investigative behavior acts this way. To analyze a whole, we must isolate it and we usually pick a whole at its boundary. We then break into it to find its parts, again at boundaries. We must relate the parts one to another and to the whole. Analyzing and investigating leads us to look for these boundaries, interactions, and levels. Put in another way, I have presented a model (in the communicative world) which revolves around wholes and parts, boundaries and interactions. When I then look at the "real world," I look for such boundaries and interactions and find them. But, in a certain sense, they were created by scientists who investigated them in this way. Is it then really so surprising?

6) Reductionism and Analysis-Synthesis, Down, Up, and Sideways

The hierarchical model helps resolve, in an interesting fashion, the problem of reductionism in the biological sciences. Reductionism can be, and has been, looked upon in two different but related ways:

In the first way, the data of the first science is explained by using the terms, properties, and theories of the second. The first science is then said to be reduced to the second.

In the second way, one explains the data on one level of organization by using the data from the level below it.

A corollary of this view is that the data from the lower level must be constrained by the data from the upper level. The first way is the way that the word reductionism is usually used scientifically. The second way should use another word, and that is best expressed by the words analysis, then synthesis. We break the whole into parts (analysis) so as to understand the whole when we put the parts back together again (synthesis). Since sciences generally are grouped vertically with one science above the next (see Figure 1), not infrequently the "lower" level science can be used to explain the upper one. This is where the two ways intersect.

The reduction problem arises because science explores nature horizontally. It is interested in elements (which are arranged horizontally) rather than subsets (which, in this model, are arranged vertically). There often is the implicit assumption that the lower level is more "basic" and better understood than the one above it. But we can not necessarily assume this. Nor are the higher levels necessarily better understood, either. What then to do? We could continue to work on

different levels with disconnected models, which would keep getting more disconnected. The better-established would bite away at the edges of the less-established, in this way coalescing like so many soap bubbles into bigger but fewer. This may be the way it goes.

What I have done in this book is to approach the reductionism problem from a different direction. That is, what properties of living systems are common to, or present on, most levels? This is a vertical rather than a horizontal approach. Approaching it this way, we do not have to be concerned with whether a level is dependent upon the one below it or not. Rather, we organize our data regarding particular properties on many levels at once, property by property. **We avoid the analysis-synthesis (down and then up) problems because we approach the problem from the side.** This is perhaps best seen in Equation 7. Naturally, as one ascends the hierarchy, more properties come into being, but this can be dealt with. Granted, I have listed only a few properties and many more would have to be added. I think, however, that this a viable approach, a new beginning to a difficult problem.

7) The Three Aspects of Our World

I have found it useful to think of our living in three worlds, or more properly, that there are three aspects to the world we live in:

1) We have the external or "real" #1 world, which I think we can safely assume exists "out there" independently and prior to observers.

2) The internal world or the world we perceive and act upon is our #2 world. This is the internal representation of the first world, both sensory and motor. Only living things possess this aspect.

3) And, finally, higher organisms are able to symbolically represent and communicate about both world #1 and world #2. This is the #3 world, or the world of communication and symbolic representation.

The first world contains the second and third worlds, but the second world evolved from the first and the third from the second. Yet they are fundamentally different from one another. There may or may not be a one-to-one relationship between these three worlds, usually not.

One may look at science from these three aspects also. Generally science is interested in investigating world #1. Scientists do this by observing world #1 and designing experiments to investigate it. This is done in world #2. However, what is observed must be measured and communicated, and this is done in world #3. This is done with numbers, graphs, models, pictures, natural or informal languages (e.g., English), and formal languages (e.g., mathematics), etc. Once again, it should be noted that in science the second world usually does not exist without the first, and the third without the second. They evolved in that order.

It seems to me that what we do in science is first to try to describe the external #1 world using the symbols of the #3 world, and acting through the #2 world. After we have developed appropriate concepts and symbols, we then build pictures, maps, plans, and models of the first world, trying to get a good "fit" using these third world symbols. If we get a good fit, we say we have a good or true model. If not, then it is a poor or false model. Sometimes our perceptions may indicate one thing ("the world is flat"), but from other perceptions and logic we may feel that our perceptions are false ("the world is round"). This may be most evident when we build mathematical and deductive models. These third world models may connect with the first world through only a few second world points (e.g., relativity theory). Most important, one must differentiate between the third world and the first world. The word is not the thing, the map not the territory, the icon not the the person, and **the architectural plan not the house**. One must be always aware of Whitehead's fallacy of misplaced concreteness.

Our knowledge of the first world comes from three main sources:

1) From observations, manipulations, and learned knowledge of the first world (through our second and third world).

2) From instinctual, "hard-wired," or a priori knowledge of the first world (inherited from our ancestors' second through first worlds).

3) From logical and mathematical manipulation regarding the first world (through the second and third worlds).

For example, our knowledge of black holes comes from the relativistic model in the third world, not from happening to observe such a hole in the first world. To confirm our mathematical deductions, however, we must go back to the first world via the second world.

I have found it helpful to think about these matters using Figure 62 below. The horizontal lines represent the three aspects of our world. They are:

a) The external world

The external world is everything that is outside the perceiver and doer, including other perceivers. We know this only because it is perceivable, has been perceived, or logically deducible from that which can be perceived. Thus, for years scientists felt that atoms existed in the external world, and yet they were not directly perceivable. They felt confident because of multiple other perceptions that led logically to the idea of atoms.

b) The internal or perceived and behavioral world

Only living things can perceive the external world (on the sensory side of their nervous

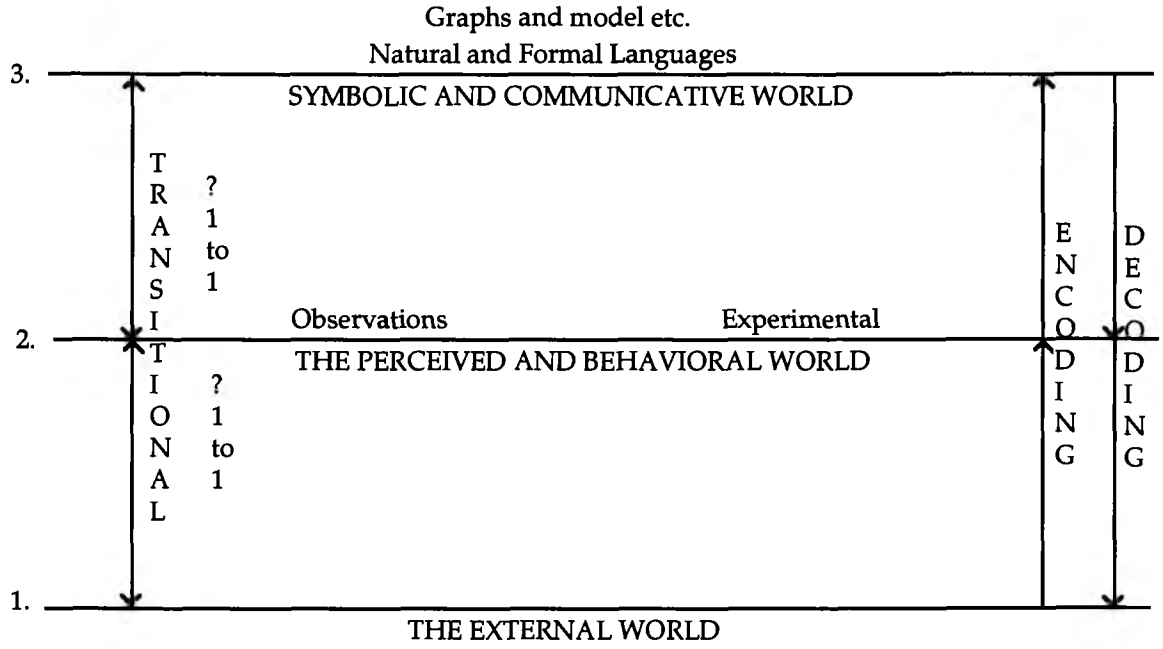


Figure 62.

system) and act upon it behaviorally (on the motor side of their nervous system). There are, of course, all gradations of perceiving and behaving, from the crude sensations and movements of an amoeba to the elaborate perceptions and behavior of human beings.

c) The symbolic and communicative world

Although present in a rather rudimentary fashion (by our standards) between animals, this is again most highly developed in human beings. Above the lines, I have shown what in these worlds is most pertinent to science.

1) I presume that everything in the external world is of interest to science and scientists, including world #2 and world #3. Of course, for a variety of reasons, scientists may not be able to study it all.

2) Scientists study the world by making observations and measurements (perceptions) and by conducting experiments, etc. (behavior).

3) Scientists communicate to themselves and others in the third world. They use everyday or informal languages (say English), and formal languages (say mathematics). Their observations and measurements are recorded and communicated in this third world, as are their hypotheses, theories, and models.

The three aspects of our world are not clearly separated, for there are transitions between them. Thus, between #1 and #2 we have my perceptions (my second world) of you and your behavior (in the first world). Between #2 and #3 we have perceptions of symbols, etc. And between #1 and #3 we have symbols and signs that are in the external world (e.g., written words, carved symbols, painted pictures, etc.). In fact, many natural world #1 things may be world #3 symbols (dove-peace, snake-evil, etc.). Sometimes there may be a one-to-one relationship between these three worlds. That is, I point to that round thing on the wall with hands and numbers and say "that clock." Far more often there is no such relationship. Language may be very abstract and is often meaningless, having no representation in the second or first world. And as pointed out earlier, perceptions may also often be misleading.

The point of all this is that it is important to realize that the scientist's third world, his theories and models, is different from his first world, or the world that he is investigating. The models are wholly human, made of symbols, while the external world is usually made of harder stuff. We speak of the model being true if there is a good agreement (1-to-1) between the symbols and the thing-in- itself, it being false if there is not. However, there can never be an absolutely "true" theory or model, since they are made of different substances. Thus, I think that perhaps it is better to speak of good or poor models, depending upon the degree of congruence or the closeness of fit. Truth has binary connotations, that is, true or false, all or none. Goodness or badness is more quantitative, since a scale is implied. Of equal importance is the fact that a model implies that there can be a variety of symbolic models modeling the same thing in the external world (as in the wave and particle models of light). If we confuse our models with the external world, we get into conceptual problems. How can the external world be both this and that? With mathematical models we speak of encoding our perceptions and measurements of world #1 through #2 into world #3. After our mathematical manipulations or inferences in world #3, we then decode back into world #2 and see if our inferences fit world #1.

8) Horizontal Versus Vertical Mathematical Reasoning

As mentioned in the beginning of this paper, partial differential equations have not been as helpful in investigating and ordering living systems as they have been in physical systems. I believe that the reason for this is that partial differential equations, in the hierarchical model context, deal primarily with horizontal continuous systems and not with vertical discontinuous systems. The infinitesimal "objects" that it deals with are all on the same level. When one does reach a boundary, one sets up boundary conditions, but the more boundaries one reaches, the more complicated the mathematics becomes. With more than two or possibly three boundaries or levels, I think the mathematics becomes very difficult. If one deals with "infinitesimals" such as the organelles in a living system, there are a large number of boundaries above them. On the other hand, if one deals with larger objects (say organs), one may not have as many boundaries above them, but we are hardly dealing with "infinitesimal" objects. Thus, while partial differential

equations may be very useful when dealing with horizontal continuous systems, with systems that are only one or two levels deep, when we are working with biological systems that are on many levels, when we are investigating vertically with many discontinuities, this mathematics will not do.

In my model I have used set algebra. It seems to me that other mathematical languages might also work, but I do not know what they would be. It seems likely that because of the nature of living systems, their hierarchical structure, one would need a discrete mathematics. Also, because of the difficulty in measuring many biological concepts or properties, there would be advantages in not having a mathematics based on integers or the reals.

9) The Vertical Dimension and Biological Space

I would like to consider in a little more detail the vertical dimension and the resulting biological space that I began this paper with. In the set elevation plan shown in Figure 60, one sees what I have called the vertical dimension. In this model, it can descend to quarks H(-10) and ascend to the world H(5) (also see Table I). I have called this the vertical dimension to contrast it with the horizontal dimension (the three usual dimensions). It could also be called the fifth, point, or zero dimension. Fifth because it follows the fourth, point because it is a repetitively magnified point, and zero (0) because of its association with the point. I have also called it the scaling dimension, for reasons already discussed. In contrast to the three spacial and one time dimension, it is not continuous and cannot be measured with the reals. It can be numbered with integers, however.

If one now considers Figure 59, it will be seen that the element plan allows one to place other living systems in relation to the subject and to each other. That is, it shows the biological space of the subject or organism. Biological space is different from physical space in that absolute distances are not of particular interest. Of more importance are the relationships between various objects or systems. The physical spaces in various organisms may be completely different from each other, while their biological spaces may be quite similar (both mice and elephants are mammals).

Upon inspection of these plans, once again it becomes apparent why differential equations are so infrequently useful in describing biological systems. Biological space is not homogeneous like physical space. It is filled with boundaries and discontinuities, making the description of more than two or three levels using differential equations much too difficult. The continuity equation of mathematical physics is one of its most important and reflects the assumption that physical space is usually continuous. But biological space is discontinuous and the continuity equation is not appropriate. I would like to think that the biospace or "discontinuous" equation is.

It is instructive to compare hierarchy space with Euclidian space. To begin with, they are both **models** of reality and are not to be confused with the thing in itself. In Euclidian space, the point is indivisible, the line has no breadth, and the plane has no thickness. Furthermore, the parallel postulate holds. But, of course, in reality, all points are divisible (can be magnified) and all lines have breadth and planes thickness. And the parallel postulate may not hold. In Euclidian space, there are three dimensions, the first associated with a line, the second associated with a plane, and the third associated with space. Hierarchy space, on the other hand, has four dimensions. The first three are as above. The fourth is associated with the point and because of this might be called the 0 dimension. The point is divisible, and how "big" or "small" it is depends upon one's vantage point. One applies different scales on different levels, and, as seen earlier, I have called it the scaling dimension.

The idealized Euclidian space model fits reality well for small distances, but for very large distances (galactic or larger), it may not. Better fits may be made by one of the non-Euclidian space models. On the other hand, biological space is different for each biological system. Different biological systems have different boundaries and interactions that define different levels, which, in turn, define different vertical dimensions and, therefore, space models.

IV) SUMMARY

As my thinking has evolved, I have gone from a rather narrow goal to a much broader one. Originally I had hoped to throw some light on some of the psychosomatic and scientific field interface problems that physicians face. I had hoped to do this using mathematics. As my thinking progressed, however, I found that perhaps I might be able to set up a broadly based, far ranging deductive system within the life sciences. My assumption was that this was a worthwhile goal. It seemed quite brash, however, to think that I would entertain this idea, until I realized that I was only dealing with a mathematical model of reality, and that many such models could exist, none of them being absolutely true. They could only be good or bad mirror images of reality. Models guide us in what to look for in the real world, and where. They are not the real world. **A formal language is no more "real" than a everyday or informal language. It is only more logical.**

As I progressed in my thinking, it became evident that I needed some overall guiding principle, and this became an analogy with **the method of elementary abstraction** that is used in mathematical physics. As I saw it, rather than abstracting infinitesimal elements from say some body of water that one wanted to describe, I was abstracting sets of discrete elements from hierarchies found in living systems that I wanted to describe. Rather than ending up with the formal language, partial differential equations, I ended up with the formal language which I called hierarchy algebra, a kin to set algebra. Rather than abstracting to an equation of continuity, I abstracted to an equation of discontinuity which I called the hierarchy space equation. Instead of inserting into this equation various other equations, I inserted various other properties (variables) of living systems, thus arriving at the hierarchy property equation. This equation H(L,I-n) could generate the universe of subsets of these properties. Instead of integrating back to testable entities, I substituted in my equation the values that were reported by others (the data) and solved for H(L,S-n), that subset that described the living system using these elements and properties. Thus, I started with concrete elements and some of their properties, moved to some abstract equations and models, and then came back to the concrete, now organized however in a very specific way, namely hierarchically. I called this **the method of abstracting the elements**.

The language and model that I have built is structured around the hierarchical nature of living systems, the fact that they seem to have levels of organization. I have used set or Boolean algebra as my mathematical vehicle and three properties of living systems to define these levels. These are:

1) The fact that many elements from a lower level seem to go into making an element on the level above. I have called this the many-to-one property. Two other properties of living systems make this possible.

2) The first is that a boundary or container may collect together the many elements, this subset forming a new element at a higher level.

3) The second is that elements may be all "tied together" by interactions between them, this subset also becoming one element at the higher level. Usually we find all three properties present. I then developed some simple visual models (architectural renditions) which I called set and element floor and elevation plans. Pulling all of this together I arrived at a model which I have come to call the hierarchy space equation. Just as the continuity equation helps define physical and "horizontal" space; so this helps define biological and "vertical" space.

Using this hierarchy model and equation, I then applied it to a number of different living systems, abstracting out of each of them there various parameters and a given property, inserting this into the equation and looking at this property vertically. That is, I have tried to determine whether the property was present or not on each level of organization. As one describes vertically

more and more properties of a given system, one gets a more and more adequate description of the system as a whole.

Because of this descriptive ability it was useful in organizing hierarchically biological data and in classifying living systems. This I felt had a unifying effect for the biological sciences. Fields that might be widely separated were now brought together because they were cut across vertically. This then lead rather naturally to some ideas on how the model might guide one in thinking about evolution and in making predictions. It helped to clarify my thoughts on the goal that I started out with, field interface and Aunt Jenny's problems. It also guided me fruitfully in some more philosophical thoughts, thoughts that seem clearer to me now then before.

It seems to me that I have achieved by goal, that of setting up a broad deductive system or formal language within the life sciences. How good or bad this is, only time will tell. As a language or instrument it can be used on an almost unlimited number of living systems. In the present book I have just scratched the surface. Obviously one can add to the model, making models that are bigger, having more properties, more descriptive power, broader hierarchical organization of data, and more predictions. Just as obviously however, I see that no such model will ever be complete. One can add properties (and their complements), but this is never ending.

The strength of the model is also its weakness. Many important biological properties are not measurable using integers or the reals. However, they are measurable using hierarchical algebra because we have only four values (subsets) of each property to deal with. But because of this the measurements are very coarse, there are no "shades of gray" and dynamically values change stepwise rather than continuously. Partially offsetting this disadvantage is the fact that many variables are being valued in the same equation so that, for instance, with a five-level, three-property, H(5,I-3) system there are eleven variables, each with four possible values.

It is evident that hierarchy algebra with its equations and set and element floor and elevation plans could fruitfully be set up in a computer program. With its mathematical capabilities and graphics, it could simulate rather elaborate architectural plans of reality. I have started upon this although much remains to be done.

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VI) APPENDICES

A) INTERPRETATION OF H(*l*), B(*l*), AND I(*l*) EOR FIVE SUPERSYSTEMS, where *l* = 1,2,3,4.

1) The Ecological system
   - H(5) Biogeographic regional system.
   - B(4) The biogeographic regional boundary.
   - I(4) Interactions between ecosystems.
   - H(4) Ecosystems.
   - B(3) Ecosystem boundaries.
   - I(3) Interactions between populations.
   - H(3) Populations.
   - B(2) Populations boundaries.
   - I(2) Interactions (bonds) between families.
   - H(2) Families.
   - B(1) Family boundaries (burrows,nests,territories,etc.)
   - I(1) Interactions between organisms.
   - H(1) Organisms.

2) The Social system.
   - H(5) The Nation.
   - B(4) The nations boundary (border).
   - I(4) Interactions between communities.
   - H(4) Communities.
   - B(3) Community boundaries (limits).
   - I(3) Interactions between extended families.
   - H(3) Extended families.
   - B(2) The extended family boundaries (compounds).
   - I(2) Interactions between nuclear families.
   - H(2) Nuclear family (household).
   - B(1) Nuclear family boundaries (homes).
   - I(1) Interactions (bonds) between family members.
   - H(1) Family members (persons,humans).

3) The Human system.
   - H(5) Person or human.
   - B(4) Boundary (skin) of the person.
   - I(4) Interactions between organ systems.
   - H(4) Organ systems.
   - B(3) Boundaries of organ systems.
   - I(3) Interactions between organs.
   - H(3) Organs.
   - B(2) Boundaries (epitheliums) of organs.
   - I(2) Interactions between tissues.
   - H(2) Tissues.
   - B(1) Boundaries (membranes) of tissues.
   - I(1) Interactions between cells.
   - H(1) Cells.

4) Organismal systems, a classification by number of levels using the cell as the base.
- H(5) 5 cellular level organism (e.g. Mammals)
- H(4) 4 cellular level organism (e.g. Round worm)
- H(3) 3 cellular level organism (e.g. Flatworm)
- H(2) 2 cellular level organism (e.g. Algae)
- H(1) 1 cellular level organism (e.g. Protozoa)

5) The cellular supersystem.
- H(5) The Cell.
- B(4) The cell boundary (membrane or wall).
- I(4) Organelle interactions.
- H(4) Organelles.
- B(3) Organelle boundaries (membranes).
- I(3) Interactions between macromolecules.
- H'(3) Macromolecules.
- B'(2) No macromolecular boundaries.
- I(2) Interactions (bonds) between molecules.
- H'(2) Molecules.
- B'(1) No molecular boundaries.
- I(1) Interactions (bonds) between atoms.
- H(1) Atoms.

6) The Atomic or molecular supersystem.
- H'(4) A molecule.
- B'(3) No molecular boundaries.
- I(3) Interactions (bonds) between atoms.
- H(3) The atom.
- B(2) Atom boundaries (electron shell).
- I(2) Interactions between nucleons.
- H(2) Nucleons.
- B(1) Nucleon boundaries ("bags").
- I(1) Interactions between quarks.
- H(1) Quarks.

B) MULTIPLYING OUT THE PRELIMINARY CELL EQUATION TO ARRIVE AT H(5,S) IN DISJUNCTIVE NORMAL FORM.

$$
\begin{aligned}
H(5,S) &= B(4) \cap [I(4) \cup I'(4)] \cap \\
H(4) \cup H'(4) &= [B(3) \cup B'(3)] \cap [I(3) \cup I'(3)] \cap \\
H'(3) &= B'(2) \cap [I(2) \cup I'(2)] \cap \\
H'(2) &= B'(1) \cap [I(1) \cup I'(1)] \cap H(1).
\end{aligned}
$$

We then multiply the equation out. Since there are 5 variables, we have 2 to the 5th or 32 terms in the equation. Parts of these terms are already defined. That is,

$$
\begin{aligned}
B(3)B'(2)I(2)B'(1)I(1)H(1) &= \text{organelles} \\
B'(2)I(2)B'(1)I(1)H(1) &= \text{macromolecules} \\
B'(1)I(1)H(1) &= \text{simple molecules (or molecules)} \\
I'(1)H(1) &= \text{non-interacting atoms}
\end{aligned}
$$

Multiplied out we get the following products--and interpretations:

```
   B(4)I(4)[B(3)I(3)B'(2)I(2)B'(1)I(1)H(1)]   Organelles interacting,
           [organelle with macromolecules ]    macromolecules interacting.
  B(4)I'(4)[B(3)I(3)B'(2)I(2)B'(1)I(1)H(1)]  Organelles not interacting,
           [organelle with macromolecules ]    macros interacting.
  B(4)I(4)[B(3)I'(3)B'(2)I(2)B'(1)I(1)H(I)]  Organelles interacting,
           [organelle with macromolecules  ]   macros not interacting.
 B(4)I'(4)[B(3)I'(3)B'(2)I(2)B'(1)I(1)H(1)]  Organelles not interacting,
           [organelle with macromolecules  ]   macros not interacting.
  B(4)I(4)B'(3)I(3)[B'(2)I(2)B'(I)I(1)H(1)]  Interacting macro not in an
                   [macromolecule          ]   organelle(duplicated below*)
 B(4)I(4)B'(3)I'(3)[B'(2)I(2)B'(1)I(I)H(1)]  Non-interacting macro not
                   [macromolecules         ]   in organelle
                                               (duplicated below*)
 B(4)I'(4)B'(3)I(3)[B'(2)I(2)B'(1)I(1)H(1)]  Interacting macro not in an
                   [macromolecules         ]   organelle *
B(4)I'(4)B'(3)I'(3)[B'(2)I(2)B'(1)I(1)H(1)]  Non-interacting macro
                   [macromolecules         ]   not in an organelle *
B(4)I(4)B(3)I(3)B'(2)I(2)B'(1)I'(1)H(1)     I(2)B'(1)I'(1)*-forbidden
B(4)I(4)B(3)I'(3)B'(2)I(2)B'(1)I'(1)H(1)    by definition - see below
B(4)I(4)B'(3)I(3)B'(2)I(2)B'(1)I'(1)H(1)          "          *
B(4)I(4)B'(3)I'(3)B'(2)I(2)B'(1)I'(1)H(1)   I(2)B'(1)I'(1)*-forbidden
B(4)I'(4)B(3)I(3)B'(2)I(2)B'(1)I'(1)H(1)    by definition - see below
B(4)I'(4)B(3)I'(3)B'(2)I(2)B'(1)I'(1)H(1)         "          *
B(4)I'(4)B'(3)I(3)B'(2)I(2)B'(1)I'(1)H(1)         "          *
B(4)I'(4)B'(3)I'(3)B'(2)I(2)B'(1)I'(1)H(1)        "          *
B(4)I(4)B(3)I(3)B'(2)I'(2)B'(1)I(1)H(1)           "          *
B(4)I(4)B(3)I'(3)B'(2)[I'(2)B'(1)I(1)H(1)] . Molecules in an
                      [I'(2) molecules   ]    interacting organelle
B(4)I(4)B'(3)I(3)B'(2)I'(2)B'(1)I(1)H(1)          "          *
B(4)I(4)B'(3)I'(3)B'(2)I'(2)B'(1)I(1)H(1)         "          *
B(4)I'(4)B(3)I(3)B'(2)I'(2)B'(1)I(1)H(1)          "          *
B(4)I'(4)B(3)I'(3)B'(2)[I'(2)B'(1)I(1)H(1)] Molecules in a non-
                       [I'(2) molecules   ]   interacting organelle
B(4)I'(4)B'(3)I(3)B'(2)I'(2)B'(1)I(1)H(1)         "          *
B(4)I'(4)B'(3)I'(3)B'(2)[I'(2)B'(1)I(1)H(1)] Molecules not in organelle
                       [I'(2) molecules   ]   but in the cell.
B(4)I(4)B(3)I(3)B'(2)I'(2)B'(1)I'(1)H(1)          "          *
B(4)I(4)B(3)I'(3)B'(2)I'(2)B'(1)I'(1)[H(1)] Atoms in an interacting
                     [non-interacting atoms]   organelle
B(4)I(4)B'(3)I(3)B'(2)I'(2)B'(1)I'(1)H(1)         "          *
B(4)I(4)B'(3)I'(3)B'(2)I'(2)B'(1)I'(1)H(1)        "          *
B(4)I'(4)B(3)I(3)B'(2)I'(2)B'(1)I'(1)H(1)         "          *
B(4)I'(4)B(3)I'(3)B'(2)I'(2)B'(1)I'(1)[H(1)] Atoms in a non-interacting
                     [non-interacting atoms ]  organelle
B(4)I'(4)B'(3)I(3)B'(2)I'(2)B'(1)I'(1)H(1)        "          *
B(4)I'(4)B'(3)I'(3)B'(2)I'(2)B'(1)I'(1)[H(1)]Atoms not in organelle
                     [non-interacting atoms  ] but in the cell
```

* note. I(l+1) is defined for H(l+1), not for H(l). In I(>l)B'(l)I'(l)H(l), we have, in effect, I(>l)H(1) where I(>l) is not defined for H(l). Therefore it is not allowed.

There are thirty-two possible terms that can be used in our description of a cell. Eighteen are forbidden by definition and two are duplicated, leaving twelve that have meaning and can be used. In looking at a "real cell", however, one may not find all of these situations present (i.e.some of the subsets may be empty.) However, I have not found any empty subsets in this case. The products that

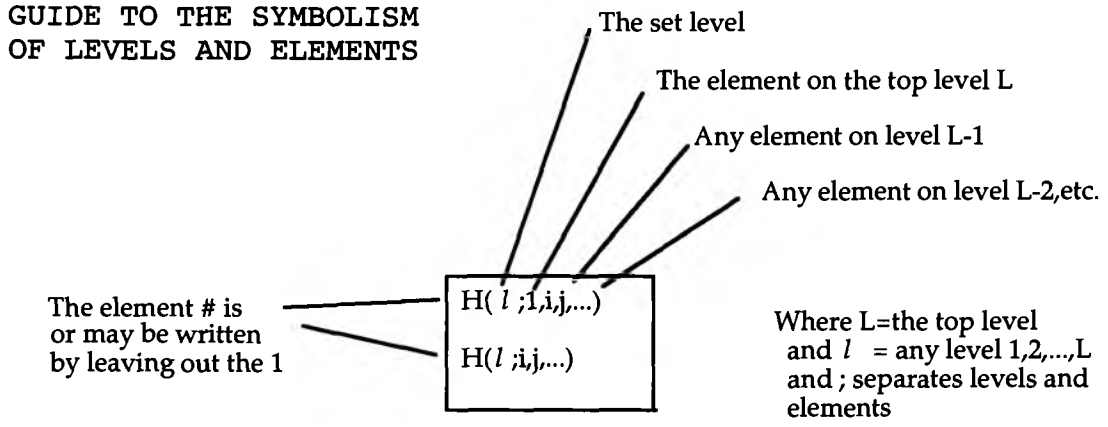
are present are then summed to give the one subset (Equation 17) that is the "set description" of the cell. The products in this description are numbered arbitrarily on the right. It should be noted that there were more than 10 to the 38th possible subsets in the universe of subsets but that only one of these gives the set description of the cell.

The set equation for a cell is then:

	level	term #
H(5)=B(4)I'(4)B'(3)I'(3)B'(2)I'(2)B'(1)I'(1) ∩[H(1)] ∪	1	1
B(4)I'(4) ∩[B(3)I'(3)B'(2)I'(2)B'(1)I'(1)]∩[H(1)] ∪	4-1	2
B(4)I(4) ∩ [B(3)I'(3)B'(2)I'(2)B'(1)I'(1)]∩[H(1)] ∪	4-1	3
B(4)I'(4)B'(3)I'(3)B'(2) ∩[I'(2)B'(1)I(1)H(1)] ∪	2	4
B(4)I'(4) ∩[B(3)I'(3)B'(2)] ∩[I'(2)B'(1)I(1)H(1)] ∪	4-2	5
B(4)I(4) ∩[B(3)I'(3)B'(2)] ∩[I'(2)B'(1)I(1)H(1)] ∪	4-2	6
B(4)I(4)B'(3)I'(3) ∩[B'(2)I(2)B'(1)I(1)H(1)] ∪	3	7
B(4)I(4)B'(3)I(3) ∩[B'(2)I(2)B'(1)I(1)H(1)] ∪	3	8
B(4)I'(4) ∩[B(3)I'(3)] ∩[B'(2)I(2)B'(1)I(1)H(1)] ∪	4-3	9
B(4)I(4) ∩[B(3)I'(3)] ∩[B'(2)I(2)B'(1)I(1)H(1)] ∪	4-3	10
B(4)I'(4) ∩[B(3)I(3)] ∩[B'(2)I(2)B'(1)I(1)H(1)] ∪	4-3	11
B(4)I(4) ∩[B(3)I(3)] ∩[B'(2)I(2)B'(1)I(1)H(1)]	4-3	12

C) A PARTIAL LISTING OF THE ORGAN SYSTEMS, ORGANS AND TISSUES OF HOST MAMMALS WITH THEIR HIERARCHICAL CLASSIFICATION NOTATION.

In classifying the mammalian system levels (organ systems, organs, tissues, and cells), the sets and elements of each, I have followed the conventions of this paper which I briefly review below. The top set system level H(5) is that of mammals and has only 1 element, the organism under consideration. Organ systems have a set system level of H(4) and any one element is H(4;i), where i = 1,2...10, there being 10 organ systems. Organs have a set systems level number of H(3) and any one element (organ) is H(3;i,j) where i is the organ system that the organ j is in. The number of organs in any organ system is n(3;i), this varying from system to system. One then works down in a similar fashion to arrive at the set system level and element level numbers for any tissue, k = 1,2...n(2;i,j) and any cell, 1 = 1,2...n(1;i,j,k). It will be noted that I have not included the subset number S which further defines the mammal using the given properties.



The numbering of the elements in each set level is entirely arbitrary and does not follow any particular order except that I have, when possible, numbered from "proximal" to "distal". Many organs and most tissues have not been entered, and no cells have been, primarilyfor space reasons. In this data bank I am primarily interested in getting the concept across. Organ systems and organs are listed beginning this page. Organs and tissues begin on page 107.

Organ system	Notation	"Organ"	Notation
1) Cardiovascular system	H'(4;1)	Heart	H(3;1,1)
		Aorta	H(3;1,2)
		Vena cava	H(3;1,3)
2) Endocrine system	H'(4;2)	Hypothalamus	H(3;2,1)
		Pituitary	H(3;2,2)
		Adrenals	H(3;2,3)
		Gonads	H(3;2,4)
		Thyroid	H(3;2,5)
3) Nervous system	H'(4;3)	Central NS	H(3;3,1)
		Peripheral NS	H(3;3,2)
		Autonomic NS	H(3;3,3)
4) Hematological system	H'(4;4)	Bone marrow	H(3;4,1)
		Spleen	H(3;4,2)
		Thymus	H(3;4,3)
		Lymph glands	H(3;4,4)

Organ system	Notation	"Organ"	Notation
5) Respiratory system	H'(4;5)	Naso-pharynx	H(3j5,1)
		Larynx	H(3j5,2)
		Trachea	H(3j5,3)
		lungs	H(3j5,4)
6a) Digestive system (Upper)	H'(4;6a)	Mouth	H(3;6a,1)
		Oro-pharynx	H(3;6a,2)
		Esophagus	H(3;6a,3)
		Stomach	H(3;6a,4)
		Intest (absorb)	H(3;6a,5)
		Liver (absorb)	H(3;6a,6)
6b) Digestive system (Lower)	H'(4;6b)	Intest (excrete)	H(3;6b,1)
		Liver (excrete)	H(3;6b,6)
		Gall bladder	H(3;6b,2)
		Colon	H(3;6b,3)
		Anal-rectum	H(3;6b,4)
7) Urinary system	H'(4;7)	Kidneys	H(3;7,1)
		Ureters	H(3;7,2)
		Bladder	H(3;7,3)
		Urethra	H(3;7,4)
8) Musculoskeletal system	H'(4;8)	Muscles	H(3;8,1)
		Bones	H(3;8,2)
9) Cutaneous system	H'(4;9)	Skin	H(3;9,1)
		Mucus membrane	H(3;9,2)
10f) Reproductive system (female)	H'(4;10f)	Uterus	H(3;10f,1)
		Ovaries	H(3;10f,2)
		Vagina	H(3;10f,3)
		Breasts	H(3;10f,4)
10m) Reproductive system (male)	H'(4;10m)	Penis	H(3;10m,1)
		Testicles	H(3;10m,2)
		Prostate	H(3;10m,3)

A PARTIAL LISTING OF THE ORGANS, AND TISSUES OF MOST MAMALS WITH THEIR HIERARCHICAL CLASSIFICATION NOTATION.

"Organ"	Notation	Tissue	Notation
1) Heart	H(3;1,1)	Epicardium	H(2;1,1,1)
		Miocardium	H(2;1,1,2)
		Endocardium	H(2;1,1,3)
Aorta	H(3;1,2)		
Vena cava	H(3;1,3)		
2) Hypothalamus	H(3;2,1)		
Pituitary	H(3;2,2)		
Adrenals	H(3;2,3)		
Gonads	H(3;2,4)		
Thyroid	H(3;2,5)		
3) Central NS	H(3;3,1)	Gray matter	H(2;3,1,1)
		White matter	H(2;3,1,2)
		Spinal cord	H(2;3,1,3)
Peripheral NS	H(3;3,2)	Nerve ganglions	H(2;3,2,1)
		Nerve bundles	H(2;3,2,2)
		Nerve endings	H(2;3,2,3)
Autonomic NS	H(3;3,3)	Sympathetic	H(2;3,3,1)
		Parasympathetic	H(2;3,3,2)
4) Bone marrow	H(3;4,1)		
Spleen	H(3;4,2)		
Thymus	H(3;4,3)		
Lymph glands	H(3;4,4)		
5) Naso-pharynx	H(3;5,1)		
Larynx	H(3;5,2)		
Trachea	H(3;5,3)		
Lungs	H(3;5,4)		
6a) Mouth	H(3;6a,1)		
Oro-pharynx	H(3;6a,2)		
Esophagus	H(3;6a,3)		
Stomach	H(3;6a,4)	Serosa	H(2;6a,4,1)
		Muscularis	H(2;6a,4,1)
		Mucosa	H(2;6a,4,1)
Intest (absorb)	H(3;6a,5)		
Liver (absorb)	H(3;6a,6)		
6b) Intest (excrete)	H(3;6b,1)		
Liver (excrete)			
Gall bladder			
Colon			
Anal-rectum			
7) Kidneys	H(3;7,1)	Cortex	H(2;7,1,1)
		External medulla	H(2;7,1,2)
		Internal medulla	H(2;7,1,3)
Ureters	H(3;7,2)		
Bladder	H(3;7,3)		
Urethra	H(3;7,4)		
8m) Muscles	H(3;8,1)		
8b) Bones	H(3;8,2)		

"Organ"		Notation	Tissue	Notation
9)	Skin	H(3;9,1)		
	Mucus membrane	H(3;9,2)		
10f)	Uterus	H(3;10f,1)		
	Ovaries	H(3;10f,2)		
	Vagina	H(3;10f,3)		
	Breasts	H(3;10f,4)		
10m)	Penis	H(3;10m,1)		
	Testicles	H(3;10m,2)		
	Prostate	H(3;10m,3)		