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Inducing vortices in a Bose-Einstein condensate using holographically produced light beams

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Abstract: In this paper we demonstrate a technique that can create non-equilibrium vortex configurations with almost arbitrary charge and geometry in a Bose-Einstein condensate. We coherently transfer orbital angular momentum from a holographically generated light beam to a ^{87}Rb condensate using a two-photon stimulated Raman process. Using matter wave interferometry, we verify the phase pattern imprinted onto the atomic wave function for a single vortex and a vortex-antivortex pair. In addition to their phase winding, the vortices created with this technique have an associated hyperfine spin texture.

OCIS codes: (020.1335) Atom optics; (020.1475) Bose-Einstein condensates; (020.1670) Coherent optical effects; (020.4180) Multiphoton processes; (050.1380) Binary optics; (050.1970) Diffractive optics; (050.4865) Optical vortices.

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1. Introduction

Light fields that carry orbital angular momentum (OAM), have attracted a lot of attention from researchers in fields as diverse as biophysics [1], micromechanics and -fabrication [2], astronomy [3], quantum communication [4] and the field of ultracold atoms [5]. While spin angular momentum is associated with the polarization of the light field, orbital angular momentum is associated with the spatial mode of the field. In the paraxial approximation, both angular momentum components can be measured separately [6]. It has been shown [7] that paraxial Laguerre-Gaussian laser beams, which are feasible to create experimentally, carry a well-defined orbital angular momentum associated with their spiral wavefronts. More generally, light beams that contain a phase singularity, carry OAM parallel or anti-parallel to their propagation direction. Every photon in the OAM beam is in the same quantum state and possesses either $l\hbar$ or $-l\hbar$ of quantized OAM, depending on the chirality of the spiraling wavefronts [7]. If more than one singularity is present, the total OAM of the spatial mode is determined by the phase variation along a closed curve that contains all singularities. This will be $\pm 2\pi l$, and the total OAM per photon in this case is $\pm l\hbar$.

The torque produced through momentum exchange between an OAM light beam and matter can be used to induce rotation of the so illuminated object. In biophysics and micromechanics this torque has been proposed to be utilizable in driving molecular motors and micromachines, which can at the same time be trapped at the location of the singularity, where the beam intensity has to vanish [1, 8–10]. Also, in the field of ultracold atoms, direct transfer of OAM from light to atoms in an ultracold gas cloud offers interesting possibilities [11–14].

An analogue of optical vortices can be realized in superfluids, where orbital angular momentum can only be added in the form of quantized vortices. In liquid helium, vortices have been studied experimentally since the 1960s [15]. With the achievement of Bose-Einstein condensation (BEC) in dilute atomic gases, a very controllable macroscopic quantum object is available, which has already been used extensively for further study of quantized vortex states [16–19]. In the regime where the number of vortices is much smaller than the number of atoms in the condensate, the manybody wavefunction of the BEC can be approximated using mean field theory. In this description, every particle in the condensate carries the same angular momentum, quantized in units of $\hbar$.

Bose-Einstein condensates have opened the path to studying many new regimes in vortex physics. For example, because of the analogy to quantum Hall physics, low filling factors of atoms to vortices are interesting to reach [20–24]. In this regime the number of atoms in the BEC is comparable to the number of vortices contained and mostly the lowest Landau level is occupied. Also, the Berezinskii-Kosterlitz-Thouless phase transition characterized by the binding/unbinding of thermally activated vortex-antivortex pairs in two-dimensional gases is subject of current research [25, 26].

Although the first vortices in BECs were produced by a phase imprinting technique [27], vortices are most commonly produced by stirring with an anisotropic trapping potential. This can be created for example by the use of a rotating detuned laser beam [16]. In this way, Abrikosov lattices of vortices with the same circulation direction are created. With this technique, it is not easy to create vortices with different circulation directions, non-equilibrium vortex states such as multiply charged vortices, arbitrary vortex lattices, or superpositions of states with different vorticity. However, other methods of producing vortices have demonstrated some of these possibilities [27–29].

Recently, a different approach to create vortices has been demonstrated: Orbital angular momentum was transfered in units of $\hbar$ from photons in a Laguerre-Gaussian beam to atoms in a BEC using a two-photon Raman process [11]. In this way, superpositions of vortex states could be created in a controlled manner. A Raman process addresses specific atomic spin states. It

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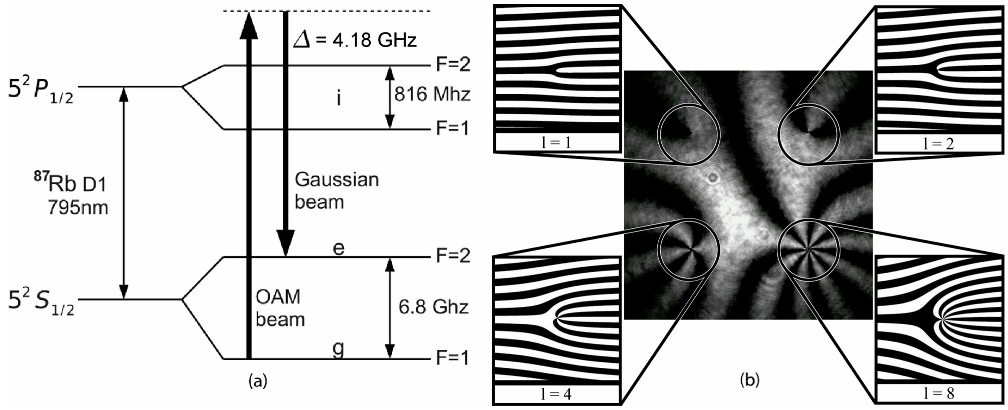

Fig. 1. (a) Raman scheme for ^{87}Rb, D1 line, using a Gaussian beam and the first order diffracted beam from a hologram. $\Delta = 4.18$ GHz is the common detuning of both beams from the intermediate $|5^2P_{1/2};F=2\rangle$ state. (b) interference pattern of a Gaussian beam with the first order diffracted beam from a hologram for producing optical vortices with $l = 1,2,4,8$ and total OAM $l = 15$. The key structures from the original binary hologram producing the respective phase windings are shown as cut outs and are related to the interference pattern. Structures in the original holograms are accurate to approximately 500 nm.

is thus possible to imprint the intensity and phase information contained in the Raman beams onto the atomic spin degree of freedom, creating a spin vortex [12–14, 30].

In this paper we present results from inducing vortices by transferring OAM from a holographically produced light beam to a ^{87}Rb BEC using a two-photon Raman process. We address the spin degree of freedom and engineer different kinds of spin vortex states, one of which is a vortex-antivortex pair.

2. Holographic production of light fields with orbital angular momentum

To be able to really take advantage of this method, one needs the ability to create OAM beams in a versatile way. Since a hologram is, in the paraxial approximation, capable of creating any desired wavefront, it can be the right tool for OAM beam creation if manufactured in good enough quality [31]. We make use of standard lithographic techniques for chrome mask manufacturing to obtain binary approximations of greyscale holograms for different OAM beams in very good quality (~ 500 nm resolution). Figure 1(b) shows cut-outs of a hologram used to produce multiply charged optical vortices as an example.

With a suitable hologram and by the use of the Raman technique, we are able to directly produce a rotational state of the BEC with a vortex-antivortex pair [29]. The coherence of the process is shown by atom interferometry, with which it can be seen that the phase of the holographically produced light beam is imprinted onto the condensate wavefunction.

Our holograms do not create Laguerre-Gaussian beams, but rather beams with a Gaussian profile except at the points of the phase singularities. The size of the optical vortices is determined by the resolution of the projection optics (about 5 μm). By using Gaussian beams with a large waist, the intensity profile away from the vortex core becomes uniform over the condensate, ensuring a homogeneous transfer rate to the vortex state. Since the beam intensity close to the phase singularity is nevertheless close to zero, atoms in the vortex core do not undergo the Raman transition. As a result, the vortices that we produce have cores filled with a different spin state. Both spin states can be imaged separately.

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3. Experimental procedure

In the experiment, the atoms start in the $F = 1$ state and are coherently transferred to the $F = 2$ state. This is done by illuminating the cloud with two copropagating beams, a Gaussian beam and a beam carrying OAM, which is one of the first diffraction orders of a hologram. The Raman scheme is shown in Fig. 1(a). An atom transferring a photon from the hologram beam to the Gaussian beam gains $l\hbar$ orbital angular momentum quanta, where l is the number of OAM quanta in the hologram beam. Since the beams are copropagating, no net linear momentum is transferred to the atoms by this process.

For our Raman lasers we chose to employ the ^{87}Rb D1 line which has fewer possible transitions paths as compared to the D2 line. The difference in frequency of the two light fields is close to 6.8 GHz corresponding to the hyperfine splitting between the initial $F = 1$ and final $F = 2$ state of the transition. σ^+ - σ^- polarizations are chosen, resulting in a change of the magnetic state by $\Delta m_F = +2$. A weak homogeneous magnetic field pointing in the same direction as the beams is applied during the Raman pulses for the definition of a quantization axis.

Our experiments begin with a ^{87}Rb BEC of approximately 3×10^5 atoms in the $|5^2S_{1/2}; F = 1, m_F = -1\rangle$ state. The condensate is in a spherical quadrupole-Ioffe configuration (QUIC) trap with a trapping frequency of $\omega = 2\pi \times 20$ Hz and has a Thomas-Fermi radius of approximately 15 μm. 14 ms after releasing the atoms from the magnetic trap, when the radius of the cloud is about 30 μm, we pulse on the two Raman beams to project the vortex pattern onto the expanded cloud. The beams, travelling horizontally, are applied as square pulses to the vertically falling cloud and change the internal atomic state to $|5^2S_{1/2}; F = 2, m_F = 1\rangle$. Beam powers of 10 μW and waists of 131 μm are used with square pulse durations of 8 μs. The common detuning of both Raman beams from the $|5^2P_{1/2}; F = 2\rangle$ state is set to $\Delta = 4.18$ GHz. Spontaneous emission during the Raman pulse duration is negligible and does not introduce decoherence relevant to the experiment.

A second Gaussian beam of the same frequency as the OAM beam is aligned copropagating with the other Raman beams. For atom interferometry two pulses are applied: the first pulse uses the hologram beam and the Gaussian beam to transfer a fraction of atoms into a vortex state. 1 μs afterwards, a second pulse is executed where the hologram beam is replaced with the second copropagating Gaussian beam at the same frequency to transfer another fraction of atoms to the same internal state, this time with a uniform phase. The pulse lengths used are 4 μs

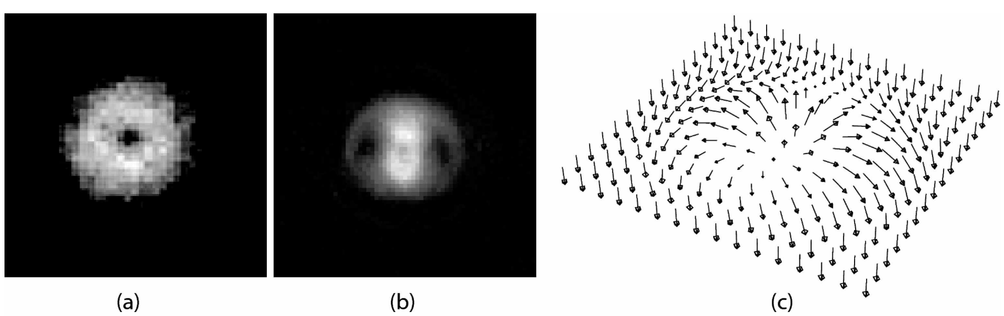

Fig. 2. (a) Absorption image of a condensate with a single, $l = 1$, vortex. (b) A vortex-antivortex state with total $l = 0$. Here, the cloud shows transfer back to the $F = 1$ hyperfine state in the center. This was chosen as a result of optimizing the Raman pulse parameters for the outer parts of the cloud. (c) A spin texture similar to a skyrmion corresponding to the core region of the vortex in (a).

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each, corresponding to $\frac{\pi}{2}$ Raman pulses (see Fig. 3(a)). Absorption images of the condensate are taken 3 *ms* after applying the Raman pulses.

4. Results

Figures 2(a) and 2(b) show absorption images of BECs with a single vortex and two counter-rotating vortices respectively. In the pictures, only atoms projected onto $|5^2S_{1/2};F=2,m_F=1\rangle$ are imaged, although a fraction of the condensate in $|5^2S_{1/2};F=1,m_F=-1\rangle$ is still present. The vortex core sizes in these pictures is limited by the resolution of the projection optics.

We emphasize here that this technique produces spin vortices [30], as opposed to stirring techniques which produce charge (density) vortices. To produce the vortex in Fig. 2(a), the intensity/pulse length is adjusted to drive a π Raman transition far enough from the singularity. If we regard $F=1,2$ as a two level system, the spin vector rotates from ($F=1$) at the center of the vortex to ($F=2$) at the edge of the core region, where the OAM beam intensity becomes constant. At the same time, the projection of the spin vector in a plane perpendicular to the direction of the beam propagation rotates azimuthally by 2π due to the phase winding in the optical vortex. This spin texture, shown in Fig. 2(c), describes a skyrmion [14]. Skrymions have been used as a model for baryonic particles in nuclear physics [32, 33]. Multi-component vortices have also been studied previously in BECs by other authors [13, 27, 34, 35].

To show the coherence of the process, the atomic vortex state is interfered with an atomic plane uniform phase state (see Fig. 3(a)). Figure 3(b) shows absorption images for different relative phases of the wavefunctions of the vortex state interfering with the uniform phase state. The phase difference between the hologram beam and the phase reference Gaussian beam maps directly onto the atoms.

In Fig. 4, we show the results of atom interferometry on a BEC state with a vortex-antivortex pair, together with simulated data. From these images, we verify that the two vortices are counter-rotating, as their phases wind in opposite directions as the phase between the light beams varies. In our experiment, we do not control the relative phases between between the two Raman pulses used in the sequence for interferometry. Thus the absorbtion images shown in Figs. 3 and 4 are to be seen as snapshots with arbitrary phases, chosen out of the various images taken.

If the hologram beam and the reference Gaussian beam used for the interference experiments are not perfectly copropogating, the structure of the original hologram can be reproduced in the

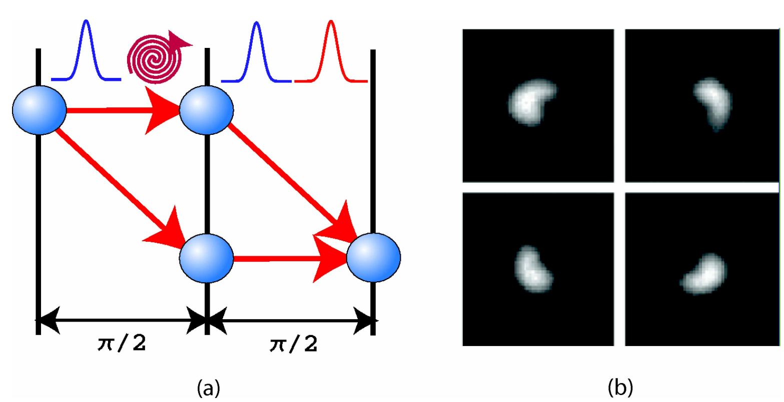

Fig. 3. (a) Pulse sequence for atom interferometry with two Raman pulses (details in text) (b) Absorption images for different phases of a single vortex, $l=1$, state of the BEC interfering with a uniform phase state.

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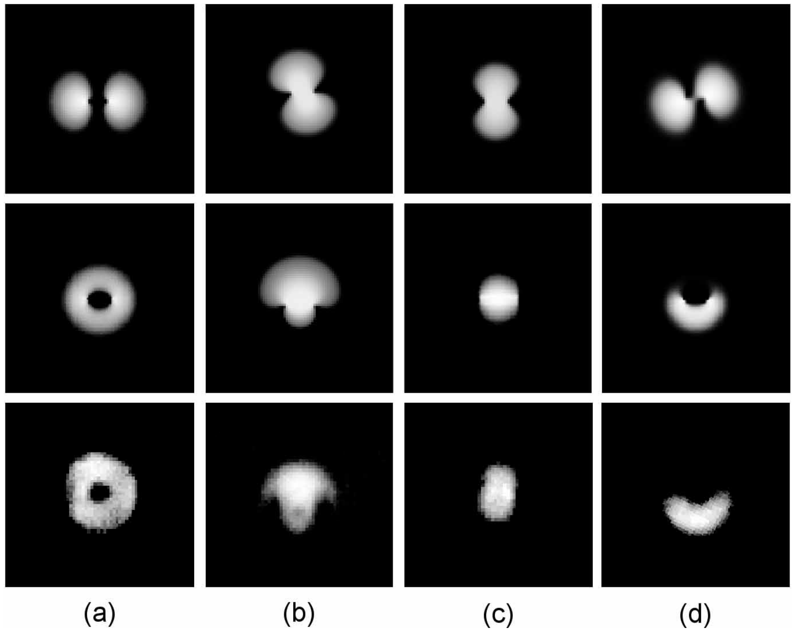

Fig. 4. Bottom row: absorption images for a vortex-antivortex state interfering with a uniform phase state. Middle row: calculated interference patterns for a vortex-antivortex state. Top row: calculated interference patterns for a state with two vortices rotating in the same direction. The simulation shows the interference patterns for phase differences of (a) $0\pi/5$, (b) $4\pi/5$, (c) $5\pi/5$ and (d) $9\pi/5$. The phases of the vortex state in the simulated data where chosen by eye in order to match the experimental data and do not resemble the exact experimental phases.

atomic cloud as an interference between the two states [36–38]. Figure 5 shows an interference image for a single vortex state obtained in this way. In this case, the misaligned Raman beam transfers linear momentum to the cloud. The atomic wavefunctions thus obtain a phase gradient with the largest component in the image plane. The interference of these, in respect to each other, tilted atomic wavefronts appears as a sinusoidally varying pattern of stripes on top of the interference pattern due to the vortex [39].

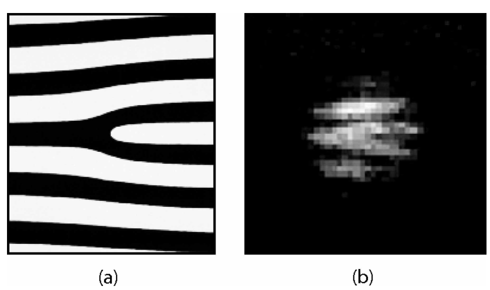

Fig. 5. A matter wave hologram (b) resembling the original hologram (a) (microscopy image of the chrome mask) is created if linear momentum is transferred in the second pulse of the described sequence.

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5. Conclusion

In this paper, we have demonstrated the controlled coherent preparation of vortex states in Bose-Einstein condensates. The phase of the classical electromagnetic field, which can be shaped holographically in a versatile way, is imprinted onto the atomic cloud, allowing great freedom in the phase pattern to be transferred. Vortex patterns with arbitrary charge, circulation and configurations can be created.

We have recently implemented high resolution optics in our apparatus (~ 600 *nm* resolution) [40]. The projection and imaging of more complex vortex patterns is now within reach. The high resolution should also enable the projection of an OAM beam containing a number of vortices comparable to the number of atoms in the BEC. Thus low filling factors in a non-equilibrium configuration are in reach, opening an avenue to studies in the quantum Hall regime. Finally, *in-situ* projection of vortex patterns would allow studying out-of-equilibrium dynamics of vortex states and addressing questions about the stability of skyrmions, merons and other topological spin excitations [31].

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