

# Supplement

## Mechanics model for vein wall distention and collapse

To study the relationship between a blood vessel volume and pressure, we analyzed the model problem of a hyperelastic cylinder subjected to internal pressure. The vena cava was modeled as a right cylinder of initial wall thickness  $H$  and radius  $R$ . When inflated, the cylinder retains its shape but changes its thickness to  $h$  and radius to  $r$ . The model assumed that the cylinder wall does not rupture and treated the wall as an incompressible, isotropic, Neo-Hookean elastic material. Due to the occlusion of the treatment zone by the distal and proximal balloons, which were connected by a relatively stiff catheter, the length of the cylinder was held constant. Shown in figure 1, the basis of  $\{\hat{e}_r, \hat{e}_\theta, \hat{e}_z\}$ ,

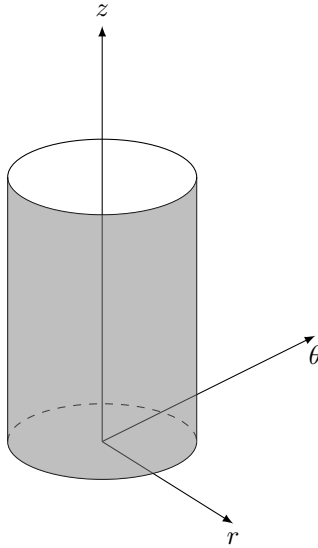


Figure 1: Cylindrical coordinate basis and cylindrical representation of the vena cava.

or the cylindrical coordinate system, will be used.

### Kinematics

The first step was to relate the changes in the radius of the cylinder to the degree that the material in the cylinder wall stretches. In examining a cross-sectional slice of the hollow cylinder, the three principal stretches ( $\lambda_r$ ,  $\lambda_\theta$ , and  $\lambda_z$ ) needed to compose the deformation gradient tensor can be written, where a principal stretch is defined as the ratio of the deformed length of a piece of material in one of the three coordinate directions ( $r$ ,  $\theta$ , or  $z$ ) to the initial length of that piece of material in that coordinate direction.

$\lambda_r$ , the stretch ratio in the radial direction, is the ratio of initial to final wall thickness:

$$\lambda_r = \frac{h}{H}. \quad (1)$$

$\lambda_\theta$ , the stretch ratio in the “hoop” (circumferential) direction, is the ratio of final to initial circumference:

$$\lambda_\theta = \frac{2\pi r}{2\pi R} = \frac{r}{R} \quad (2)$$

Finally, the stretch  $\lambda_z$  along the axis of the cylinder is unity because the length of the treatment zone is fixed by the connection between the proximal and distal balloons, the final length  $l$  and initial length  $L$  are identical, so that:

$$\lambda_z = \frac{l}{L} = 1 \quad (3)$$

To enforce the constraint that the wall of the vena cava as incompressible, we note that the initial and final volume are  $V = 2\pi RLH = 2\pi rlh$ , so that

$$\frac{2\pi rlh}{2\pi RLH} = \lambda_r \lambda_\theta = 1, \quad (4)$$

or  $\lambda_r = \lambda_\theta^{-1}$ .

### Stresses

The next step is to examine the three stresses in the wall of the cylinder:  $\sigma_r$ ,  $\sigma_\theta$ , and  $\sigma_z$ . Because the thickness of the cylinder is significantly smaller than its radius,  $\sigma_r$  can be neglected relative to the other terms so that the stress state is approximately two-dimensional. These stresses can be calculated from the “Law of Laplace” by treating the cylinder as a membrane in equilibrium with an internal pressure,  $p$ , that arises from fluid inside the vena cava or the thrombus itself, and acts uniformly on the internal lumen and both ends of the vena cava. Considering the equilibrium of a cross-section of the vena cava, the a stress  $\sigma_\theta$ , acting over the circumferential cross-sectional area  $2\pi rh$ , must balance the internal pressure acting over an area  $\pi r^2$ , so that  $2\pi rh\sigma_\theta = \pi r^2 p$ , or:

$$\sigma_\theta = \frac{pr}{2h} \quad (5)$$

A second force balance can be achieved by examining the cylinder if it were cut in half lengthwise. In this case,  $p$  acts over an area  $2rl$ , and is counteracted by  $\sigma_\theta$  which acts on the membrane walls of area  $2hl$ , so that  $\sigma_\theta hl = prl$ , or:

$$\sigma_\theta = \frac{pr}{h} \quad (6)$$

### Constitutive Behavior

Finally, the constitutive relation that relates stress to stretch of the vena cava was considered. The vena cava was modeled as isotropic and neo-Hookean, so that:

$$\sigma_\theta = 2\mu J^{-5/3}(\lambda_\theta^2 - \lambda_r^2) \quad (7)$$

$$\sigma_z = 2\mu J^{-5/3}(\lambda_z^2 - \lambda_r^2) = 2\mu J^{-5/3}(1 - \lambda_r^2) \quad (8)$$

where  $J = \lambda_r \lambda_\theta \lambda_z = \lambda_r \lambda_\theta$ . Substituting:

$$\frac{pr}{h} = 2\mu J^{-5/3}(\lambda_\theta^2 - \lambda_r^2) \quad (9)$$

$$\frac{pr}{2h} = 2\mu J^{-5/3}(1 - \lambda_r^2) \quad (10)$$

Solving:

$$1 - \lambda_r^2 = \lambda_\theta^2 - 1 \quad (11)$$

Substituting once more:

$$\frac{pr}{h} = p \frac{\lambda_\theta R}{\lambda_r H} = 2\mu J^{-5/3}(2\lambda_\theta^2 - 2) \quad (12)$$

$$p = 2\mu \frac{\lambda_r H}{\lambda_\theta R} \frac{(2\lambda_\theta^2 - 2)}{(\lambda_r \lambda_\theta)^{5/3}} \quad (13)$$

$$= 4\mu \frac{H}{R} \frac{(\lambda_\theta^2 - 1)}{(\lambda_r^2 \lambda_\theta^8)^{1/3}} \quad (14)$$

$$= 4\mu \frac{H}{R} \frac{(\lambda_\theta^2 - 1)}{(2\lambda_\theta^8 - \lambda_\theta^{10})^{1/3}} \quad (15)$$